Dynamics of Machinery

Preamble

Relation between motion and forces causing is a fascinating subject. This study is a generally referred as dynamic. Modern Engineering aims at analysing and predicting dynamics behavior of physical systems

Theory of Mechanisms & Machines is used to understand the relationships between the geometry and motions of the parts of a machine or mechanism and forces which produce motion.

TOM (M&M theory) is divided into two parts:-

1) ***Kinematics of Machinery***: Study of motion of the components and basic geometry of the mechanism and is not concerned with the forces which cause or affect motion. Study includes the determination of velocity and acceleration of the machine members

2) ***Dynamics of Machinery***: Analyses the forces and couples on the members of the machine due to external forces (static force analysis) also analyses the forces and couples due to accelerations of machine members (Dynamic force analysis)

Deflections of the machine members are neglected in general by treating machine members as rigidbodies (also called rigid body dynamics). In other words the link must be properly designed to withstand the forces without undue deformation to facilitate proper functioning of the system.

In order to design the parts of a machine or mechanism for strength, it is necessary to determine the forces and torques acting on individual links. Each component however small, should be carefully analysed for its role in transmitting force.

The forces associated with the principal function of the machine are usually known or assumed.
Ex:

a) Piston type of engine: gas force on the piston is known or assumed
b) QRM – Resistance of the cutting tool is assumed.

a & b are called static forces.

Example of other static forces are:

i. Energy transmitted
ii. Forces due to assembly
iii. Forces due to applied loads
iv. Forces due to changes in temperature
v. Impact forces
vi. Spring forces
vii. Belt and pulley
viii. Weights of different parts

Apart from static forces, mechanism also experiences inertia forces when subjected to acceleration, called dynamic forces.

Static forces are predominant at lower speeds and dynamic forces are predominant at higher speeds.

Force analysis:

The analysis is aimed at determining the forces transmitted from one point to another, essentially from input point to out put point. This would be the starting point for strength design of a component/ system, basically to decide the dimensions of the components

Force analysis is essential to avoid either overestimation or under estimation of forces on machine member.

Under estimation: leads to design of insufficient strength and to early failure.
Overestimation: machine component would have more strength than required.
Over design leads to heavier machines, costlier and becomes not competitive

Graphical analysis of machine forces will be used here because of the simplification it offers to a problem, especially in cases of complex machines. Moreover, the graphical analysis of forces is a direct application of the equations of equilibrium.

General Principle of force analysis:

A machine / mechanism is a three dimensional object, with forces acting in three dimensions. For a complete force analysis, all the forces are projected on to three mutually perpendicular planes. Then, for each reference plane, it is necessary that, the vector sum of the applied forces in zero and that, the moment of the forces about any axis perpendicular to the reference plane or about any point in the plane is zero for equilibrium.

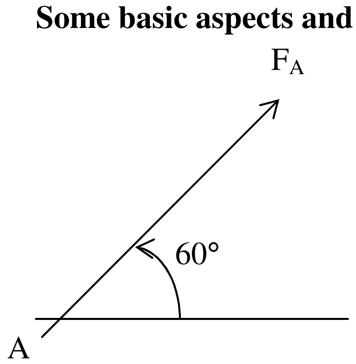
That is $\sum F = 0 \ \& \ \sum M = 0 \ or$

$$\sum F_x = 0 \ \& \ \sum F_y = 0 \ and \sum M = 0$$

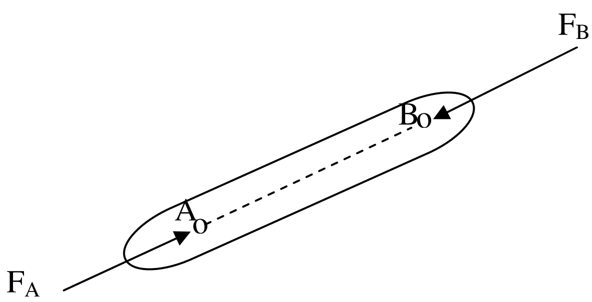
A force is a vector quantity and three in properties define a force completely;

i. Magnitude
ii. Direction
iii. Point of application

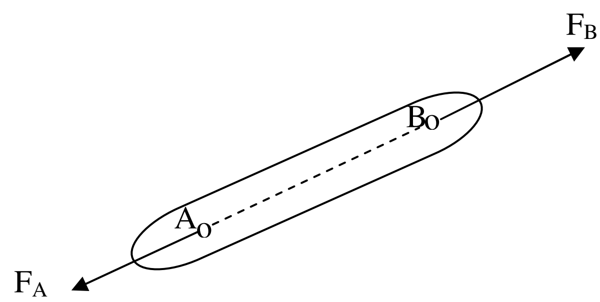
Some basic aspects and notations



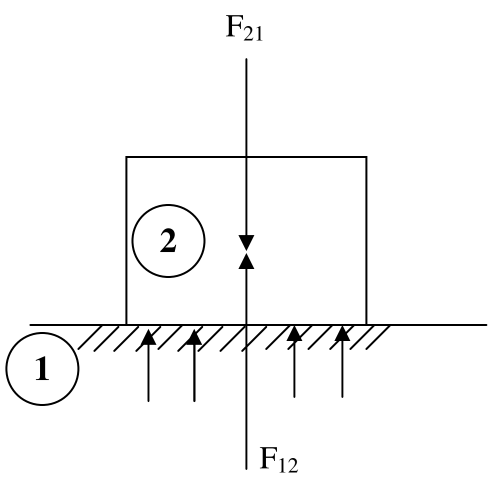
i. Force applied at A
ii. Force vector is inclined at 60° to the reference plane.
iii. Length of the vector represents the magnitude of the force to some scale.



Compressive forces



Tensile forces



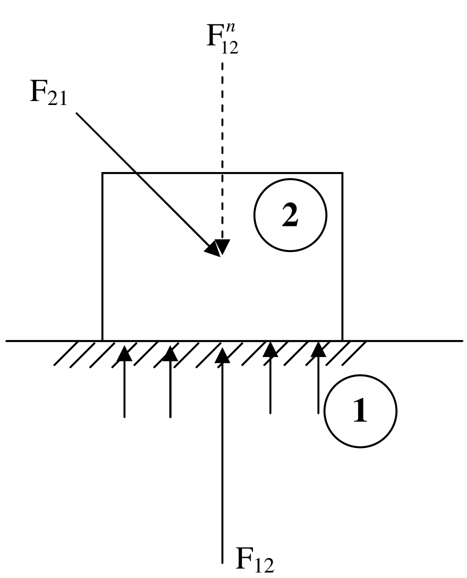
Forces are perpendicular to the line of contact

F_{21} : Force exerted by link (2) on link (1)

F_{12} : Force exerted by link (1) on link (2)

Here, $F_{21} = F_{12}$ Action and reaction are equal and opposite.

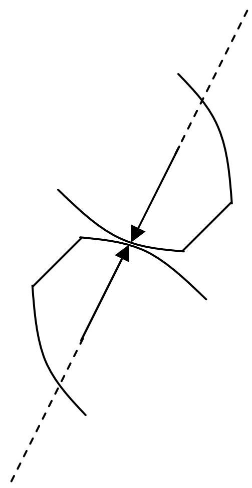
(2) is sliding on (1)



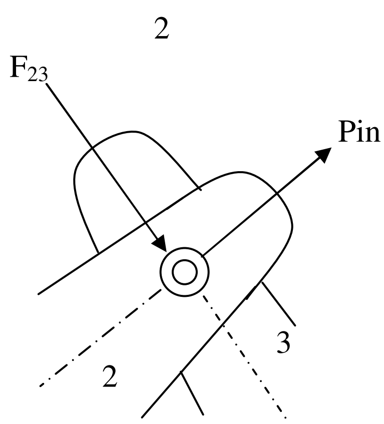
F^{n}_{12} : Vertical component of F_{21}

$F_{12} = F^{n}_{12}$

Gear meshing



Forces act along the common normal or line of action of gears.



Forces pass through the centre of the pin

Equilibrium

For a rigid body to be in Equilibrium

i) Sum of all the forces must be zero

ii) Sum of all the moments of all the forces about any axis must be zero

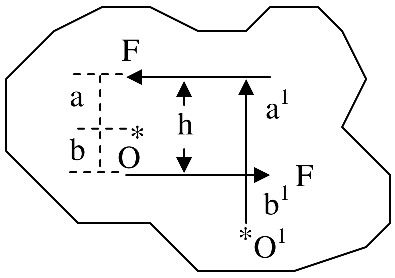
i.e, (i) $\sum F = 0$ (ii) $\sum M = 0$

or $\boxed{\begin{array}{l}\sum Fx = 0 \\ \sum Fy = 0\end{array}}$ $\sum TM = 0$, $\sum My = 0$

$\sum Fz = 0$ $\boxed{\sum Tz = 0}$ (For a planar system represented by 2D vectors)

Fx, Fy, Fz force Components along X, Y & Z axis

Similarly moments



O & O^1 are axis points

i) $Fa + Fb = F(a+b)$

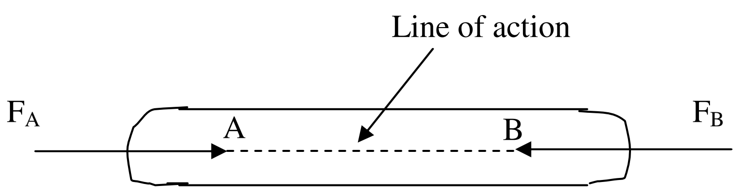
ii) $Fa^1 - Fb^1 = F(a^1 - b^1)$

(Clock wise)

"Axis point does not affect the couple"

<u>Very useful & important principles.</u>

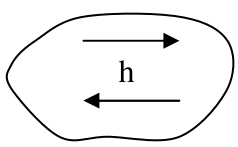
(i) Equilibrium of a body under the action of two forces only (no torque)



For body to the in Equilibrium under the action of 2 forces (only), the two forces must the equal opposite and collinear. The forces must be acting along the line joining A&B.

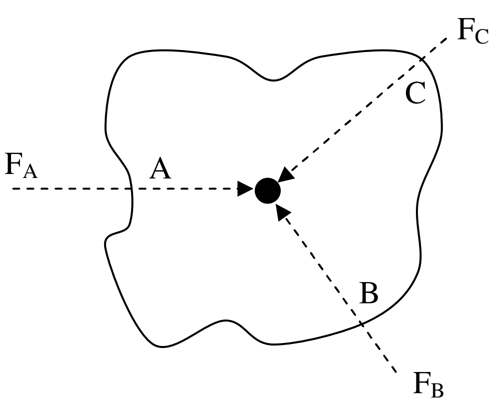
That is,

$F_A = - F_B$ (for equilibrium)



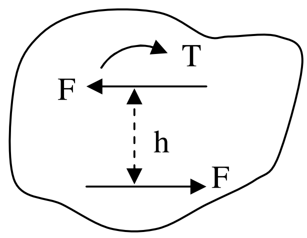
If this body is to be under equilibrium 'h' should tend to zero

(ii) Equilibrium of a body under the action of three forces only (no torque / couple)



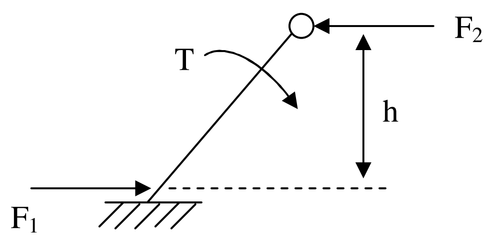
For equilibrium, the 3 forces must be concurrent and the force polygon will be a triangle.

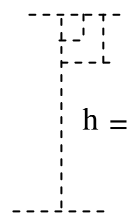
(iii) Equilibrium of a body acted upon by 2 forces and a torque.



For equilibrium, the two forces must form a counter couple. Therefore the forces must be equal, opposite and parallel and their senses must be so as to oppose the couple acting on the body

Example:





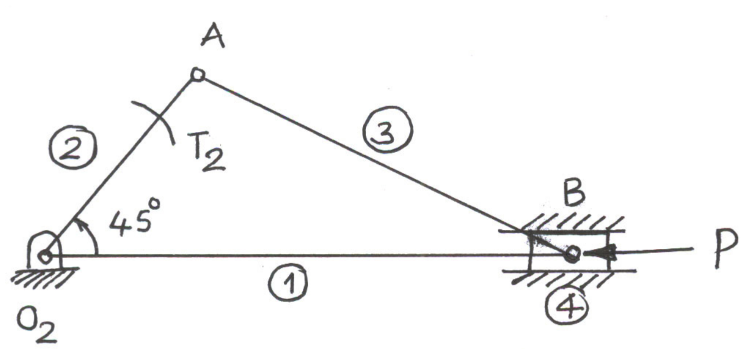
h = Perpendicular distance between F_1 & F_2

Free body diagram

The mass is separated from the system and all the forces acting on the mass are represented.

Problem No.1: Slider crank mechanism

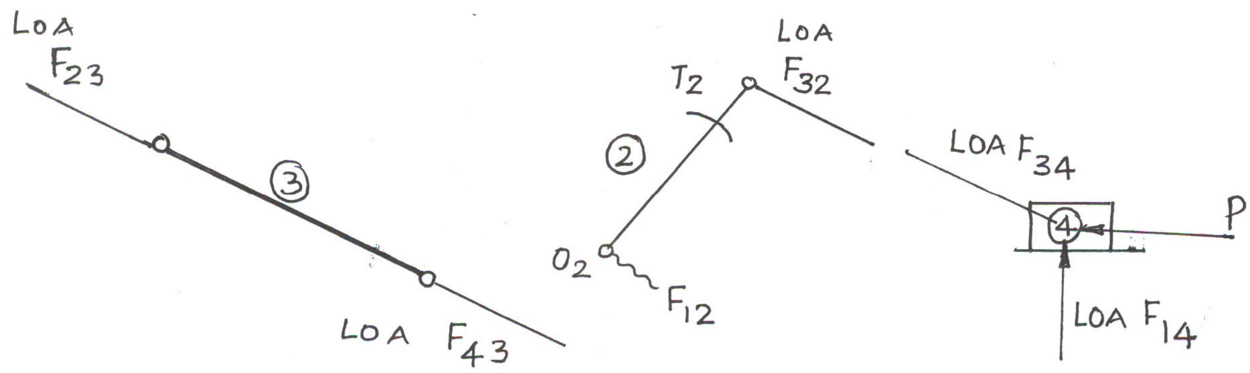
Figure shows a slider crank mechanism in which the resultant gas pressure 8 x 10^4 Nm^{-2} acts on the piston of cross sectional area 0.1 m^2. The system is kept in equilibrium as a result of the couple applied to the crank 2, through the shaft at O_2. Determine forces acting on all the links (including the pins) and the couple on 2.



$$P = (8 \times 10^4) \times (0.1)$$

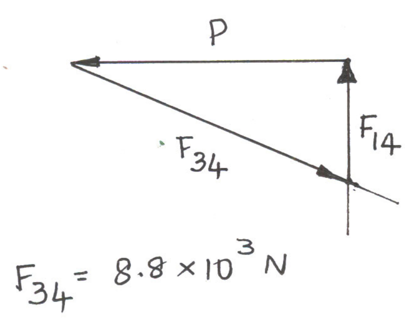
$$= 8 \times 10^3\ N$$

Free body diagram

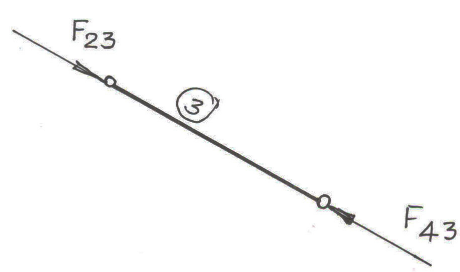


Force triangle for the forces acting on (4) is drawn to some suitable scale.

Magnitude and direction of P known and lines of action of F_{34} & F_{14} known.



Measure the lengths of vectors and multiply by the scale factor to get the magnitudes of F_{14} & F_{34}. Directions are also fixed.

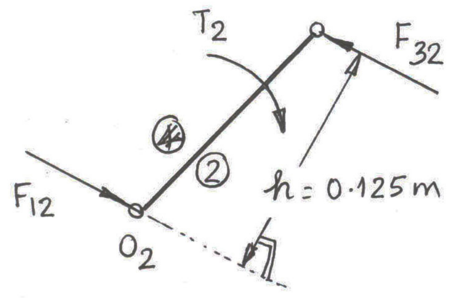


i.e, $F_{23} = -F_{32}$

Since link 3 is acted upon by only two forces, F_{43} and F_{23} are collinear, equal in magnitude and opposite in direction

$$i.e., \quad F_{43} = -F_{23} = 8.8 \times 10^3\, N$$

Also, $F_{23} = -F_{32}$ (equal in magnitude and opposite in direction).



Link 2 is acted upon by 2 forces and a torque (stated in the problem), for equilibrium the two forces must be equal, parallel and opposite and their sense must oppose T_2.

There fore,

$$F_{32} = -F_{12} = 8.8 \times 10^3 N$$

F_{32} & F_{12} form a counter clock wise couple of magnitude,

$$(F_{23} \times h) = (F_{12} \times h) = (8.8 \times 10^3) \times 0.125 = 1100 Nm.$$

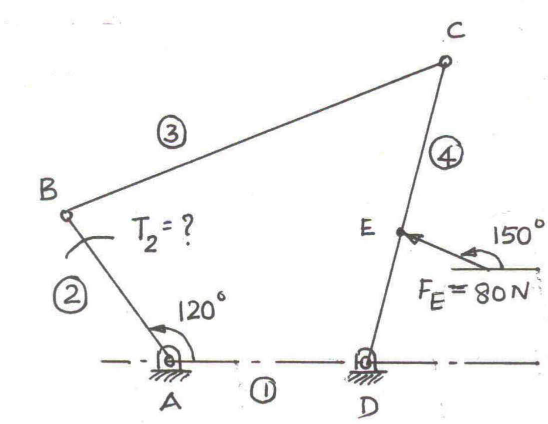
To keep 2 in equilibrium, T_2 should act clockwise and magnitude is 1100 Nm.
Important to note;

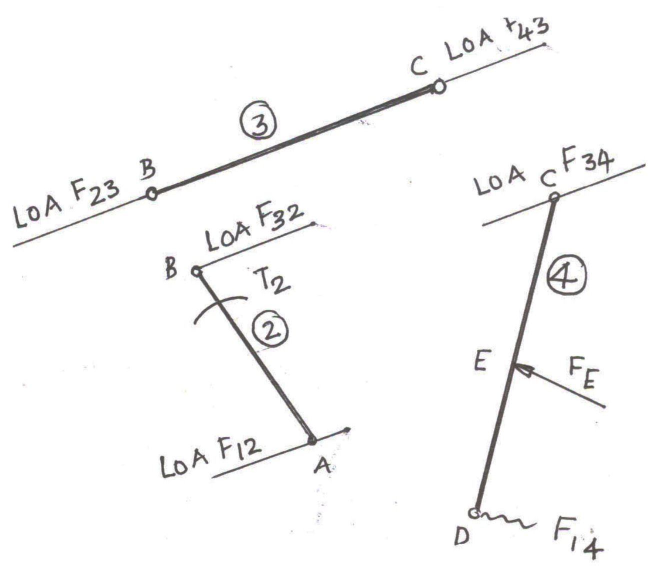
i) h is measured perpendicular to F_{32} & F_{12};
ii) always multiply back by scale factors.

***Problem No 2. Four link mechanism*.**

A four link mechanism is acted upon by forces as shown in the figure. Determine the torque T_2 to be applied on link 2 to keep the mechanism in equilibrium.

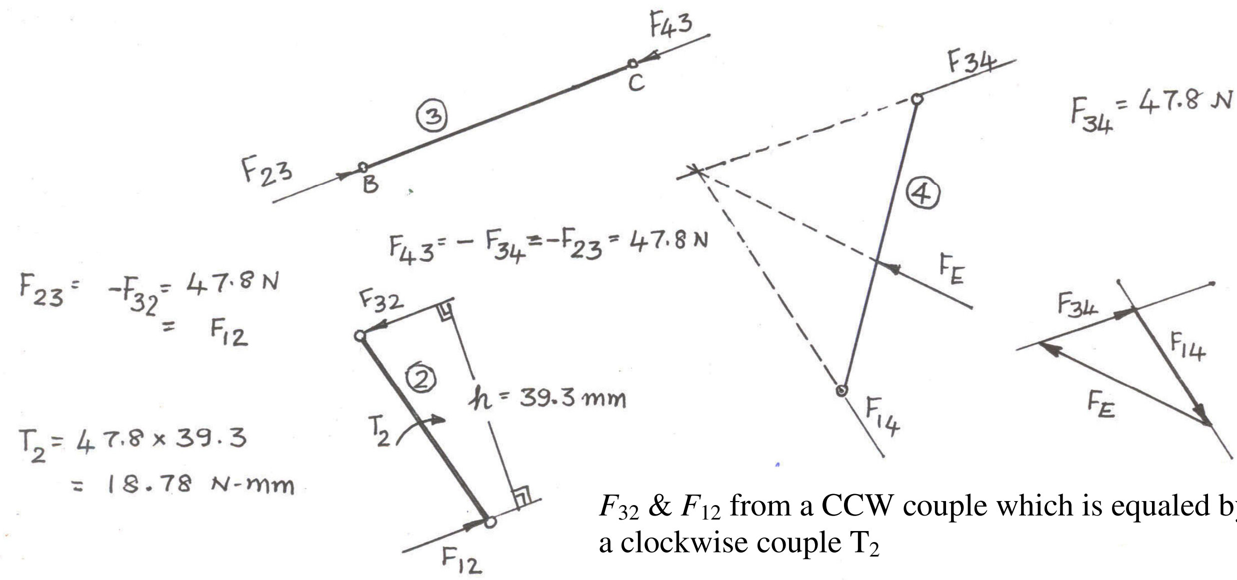
AD=50mm, AB=40mm, BC=100mm, Dc=75mm, DE= 35mm,





Link 3 is acted upon by only two forces F_{23} & F_{43} and they must be collinear & along BC.

Link 4 is acted upon by three forces F_{14}, F_{34} & F_4 and they must be concurrent. LOA F_{34} is known and F_E completely given.

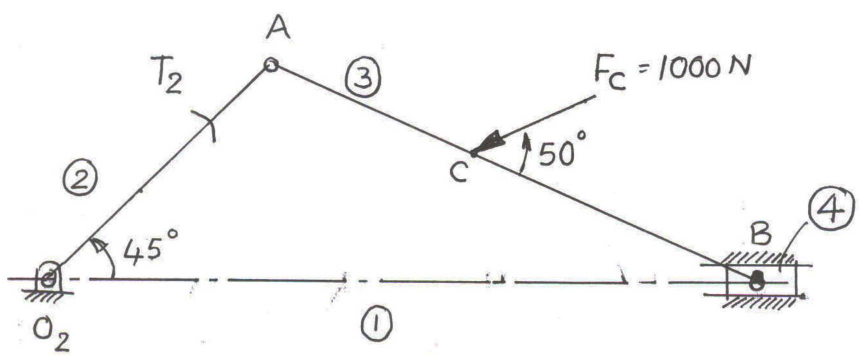


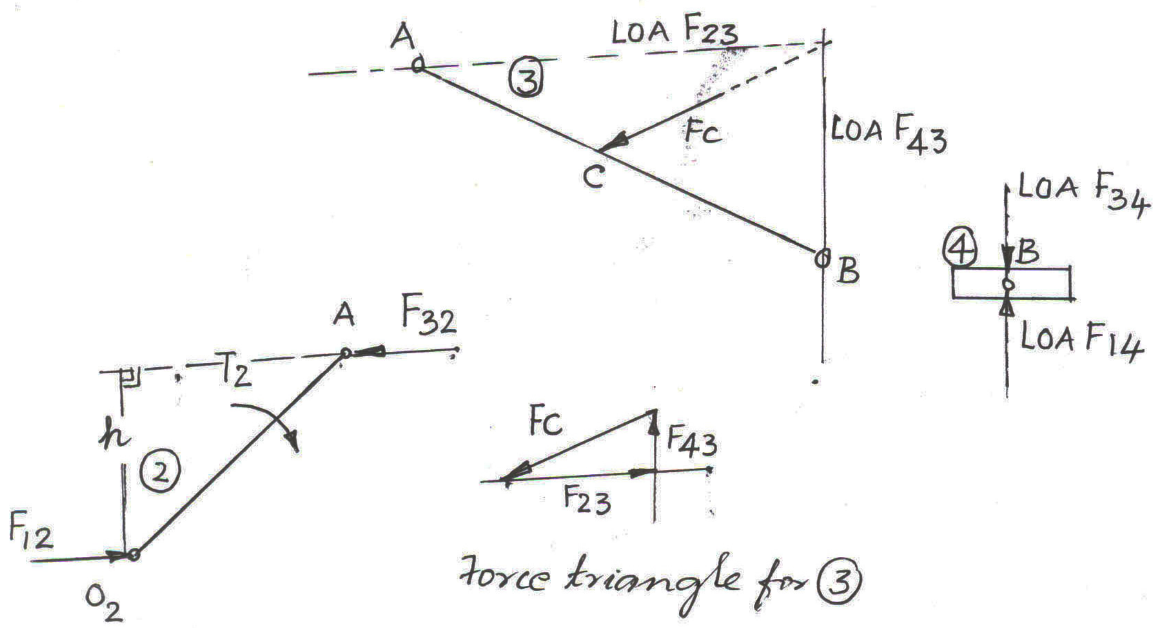
F_{32} & F_{12} from a CCW couple which is equaled by a clockwise couple T_2

Problem No 3.

Determine T_2 to keep the mechanism in equilibrium

AC=70mm,
AB=150mm,
O_2A= 40mm





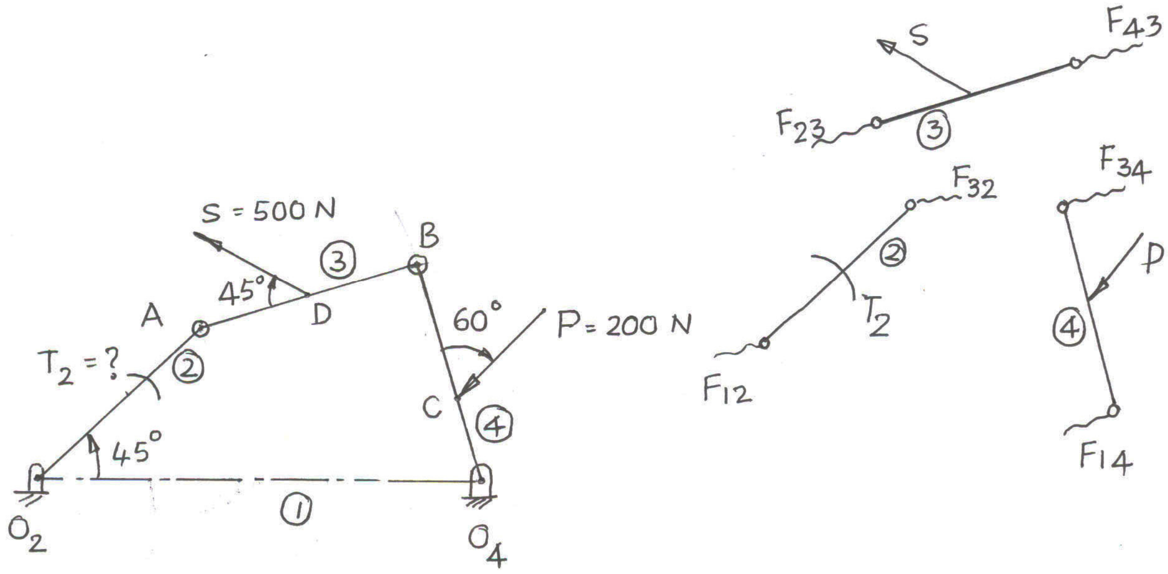
$$T_2 = F_{32} \times h = F_{12} \times h$$

F_{32} and F_{12} form a CCW couple and hence T_2 acts clock wise.

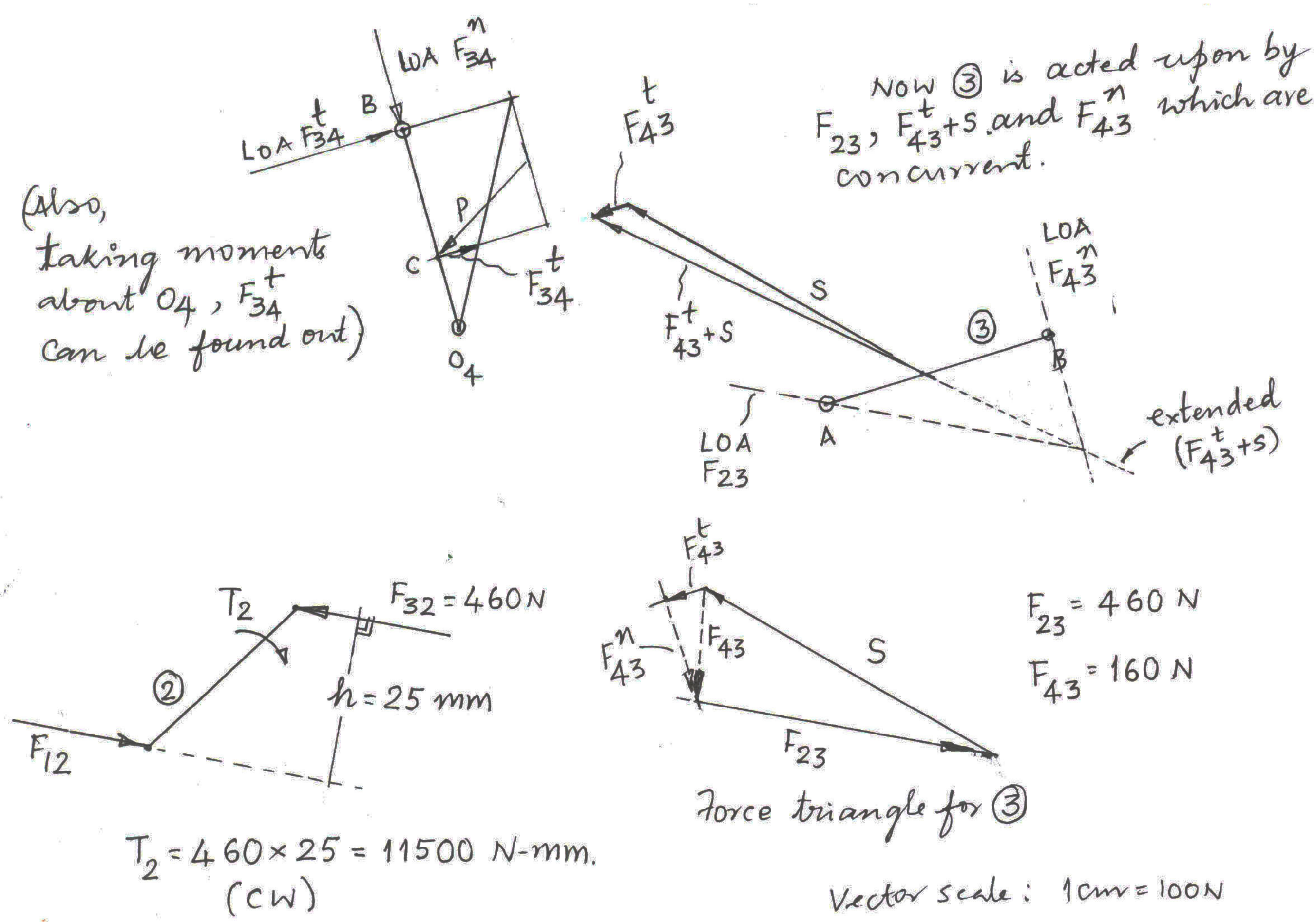
Problem No 4.

Determine the torque T_2 required to keep the given mechanism in equilibrium.

O_2A= 30mm, = AB =O_4B, O_2O_4 =60mm, $A\hat{O}_2O_4$ = 60º, BC = 19mm, AD=15mm.



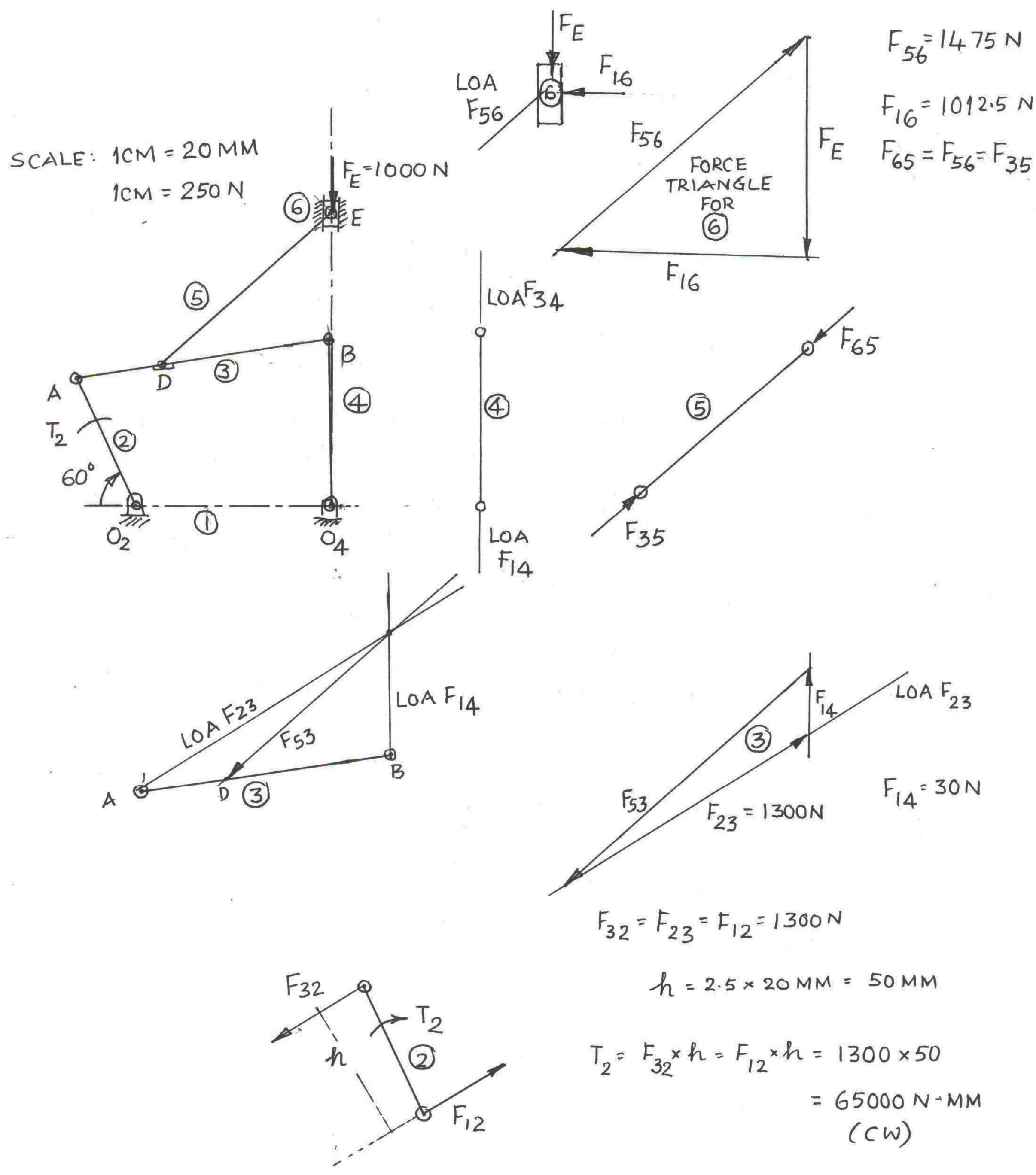
None of the links are acted upon by only 2 forces. Therefore links can’t be analyzed individually.



Problem No 5.

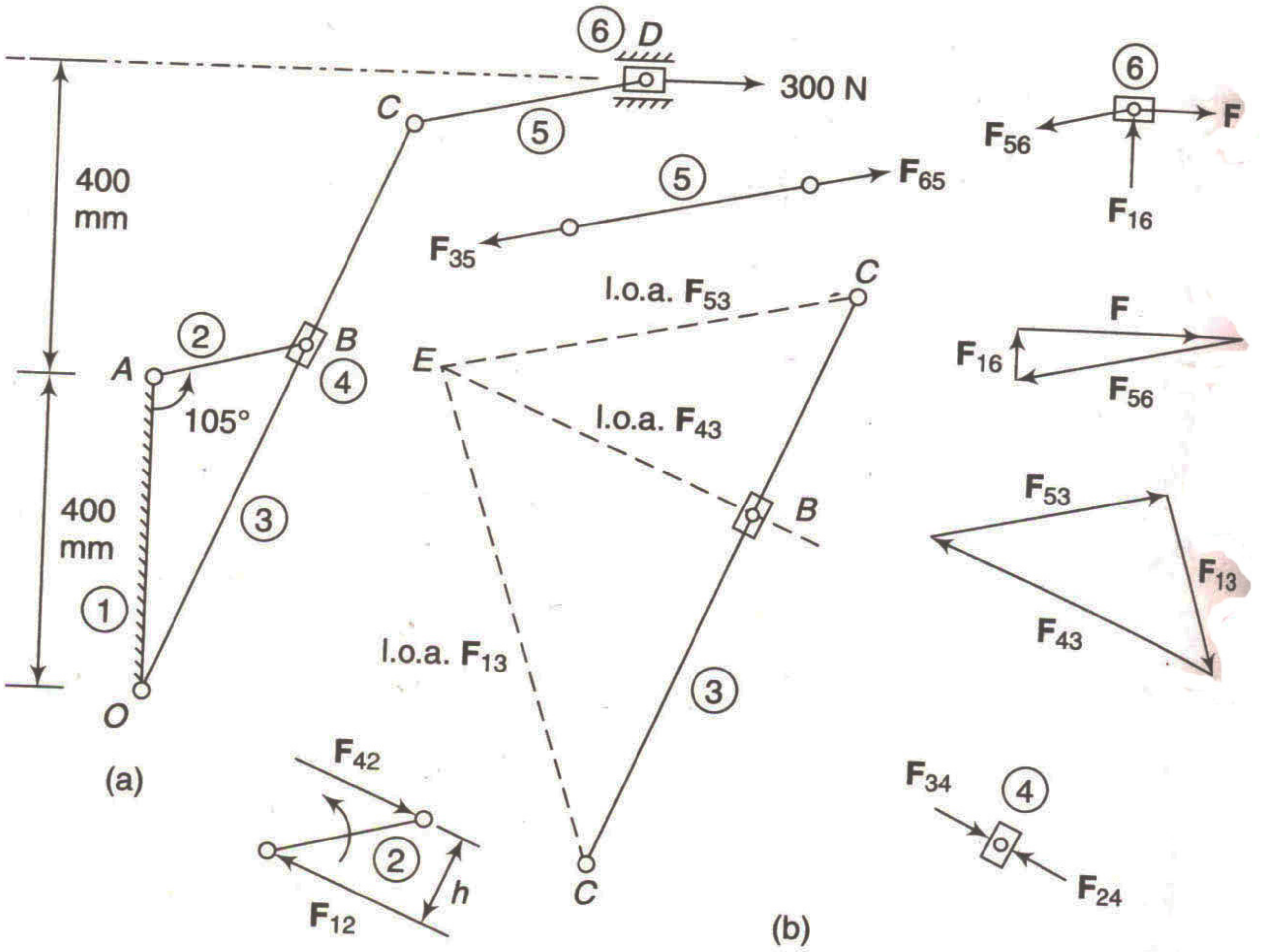
Determine the torque T_2 required to overcome the force F_E along the link 6.

AD=30mm, AB=90mm, O_4 B=60mm, DE=80mm, O_2 A=50mm, O_2 O_4 =70mm



Problem No 6

For the static equilibrium of the quick return mechanism shown in fig. 12.11 (a), determine the input torque T_2 to be applied on link AB for a force of 300N on the slider D. The dimensions of the various links are OA=400mm, AB=200mm, OC=800mm, CD=300mm



Than, torque on link 2,

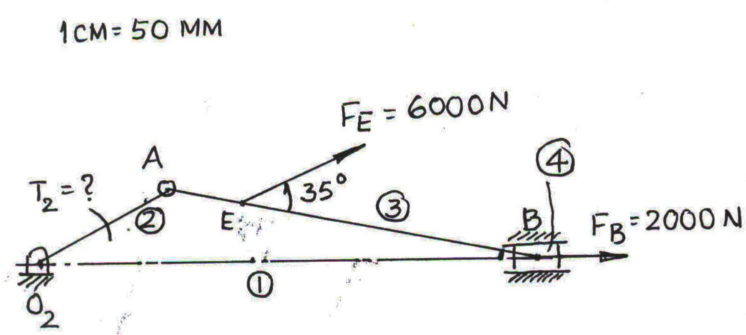
$T_2 = F_{42} x\ h = 403x120 = 48\ 360$ N counter - clockwise

Superposition method:

"When a number of forces (loads) act on a system (linear), the net effect is equal to the superposition of the effects of the individual forces (loads) taken one at a time" (Linear system: out put force is directly proportional to the input force)

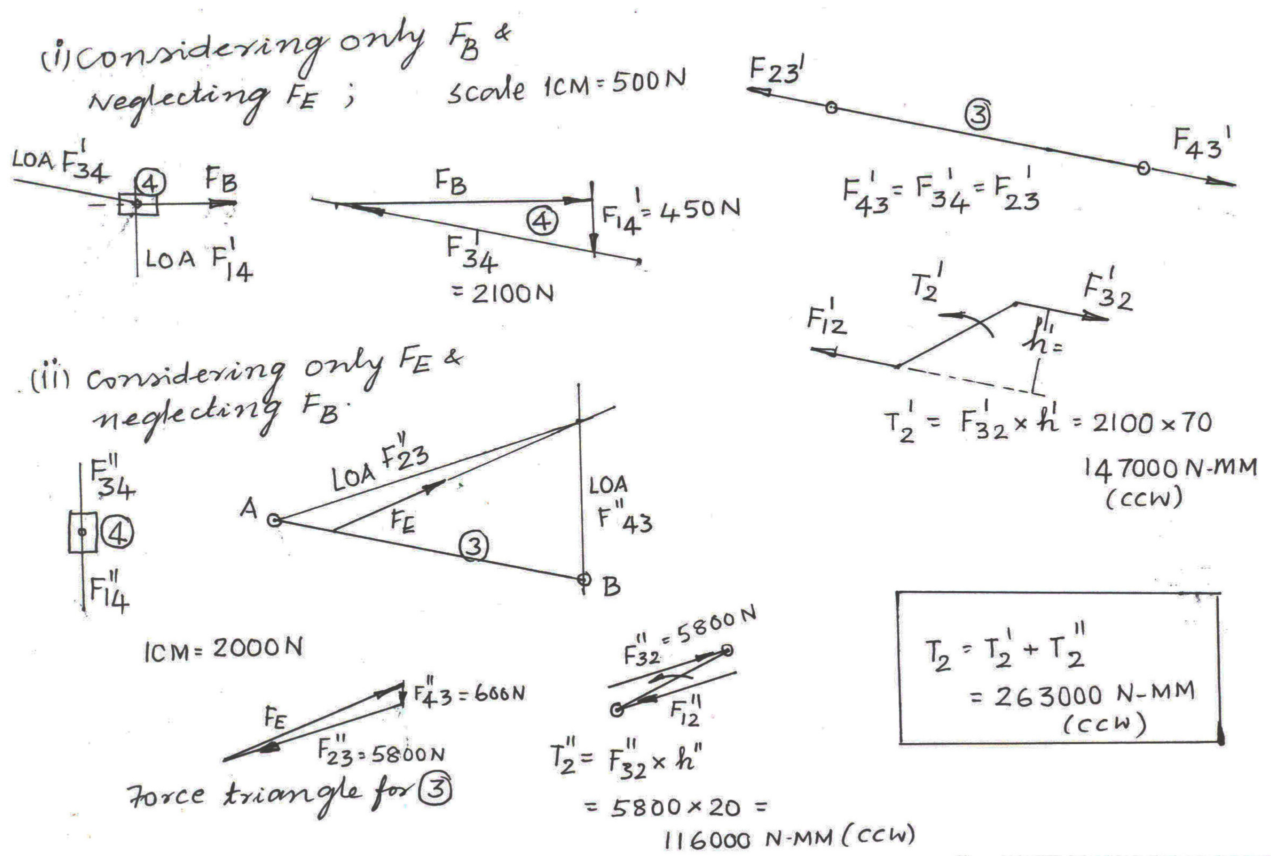
Problem No 7. Determine T_2 to keep the body in equilibrium.

O_2A =100MM, AB=250MM, AE=50MM, $A\hat{O}_2B = 30^0$



The problem is solved as two sub problems:

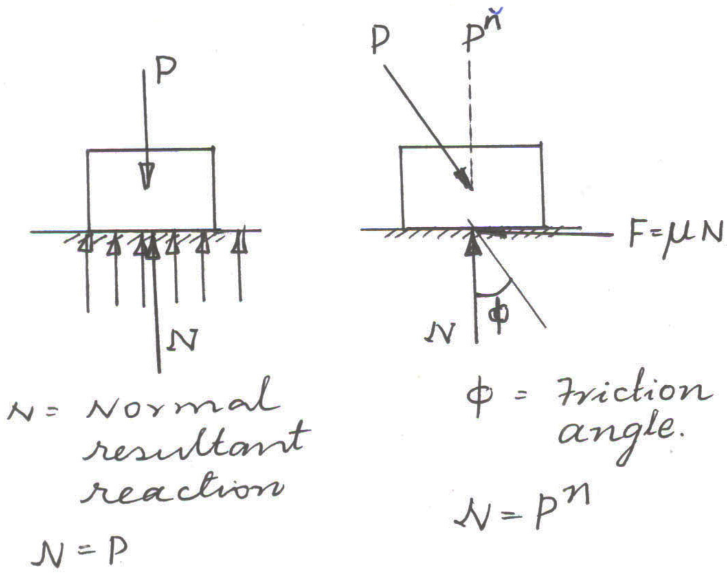
i) Considering only F_B
ii) Considering only F_E



Force Analysis considering friction.

If friction is considered in the analysis, the resultant force on a pin doesn't pass through the centre of the pin. Coefficient of friction μ is assumed to the known and is independent of load and speed.

Friction in sliding member.

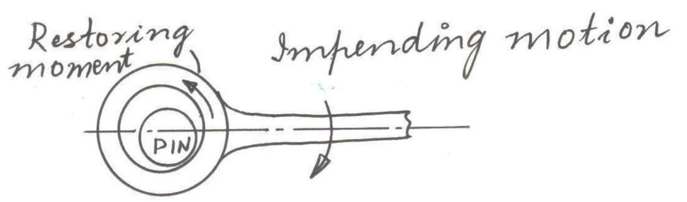


F = Frictional force

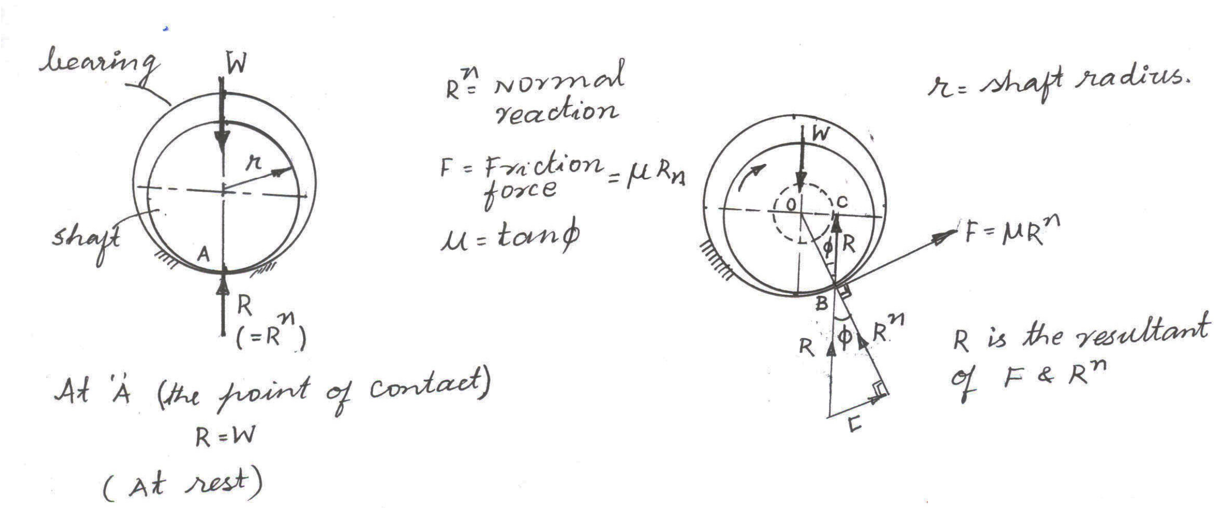
μ = coefficient of friction

$$\tan\phi = \mu = \frac{\mu N}{N}$$

Friction at pin points (bearings) & friction circle.



When a shaft revolves in a bearing, some power is lost due to friction between surfaces.



While rotating, the point of contact shifts to B; R^n passes through B.

The resultant 'R' is in a direction opposite to ω.

The circle drawn at O, with OC as radius is called 'FRICTION CIRCLE'

For the shaft to be in equilibrium; W = R

$$
\begin{aligned}
\text{Frictional moment } M &= R \times OC \\
&= W \times OC \\
&= W \times r \sin \phi \\
&= W \times r \tan \phi \\
&(\sin \phi \approx \tan \phi, \text{ for small } \phi) \\
\text{i.e, } M &= w \times r \times \mu
\end{aligned}
$$

∴ Radius of the friction circle (OC) = μr.

The friction circle is used to locate the line of action of the force between the shaft (pin) and the bearing or a pin joint. The direction of the force is always be tangent to it (friction axis)

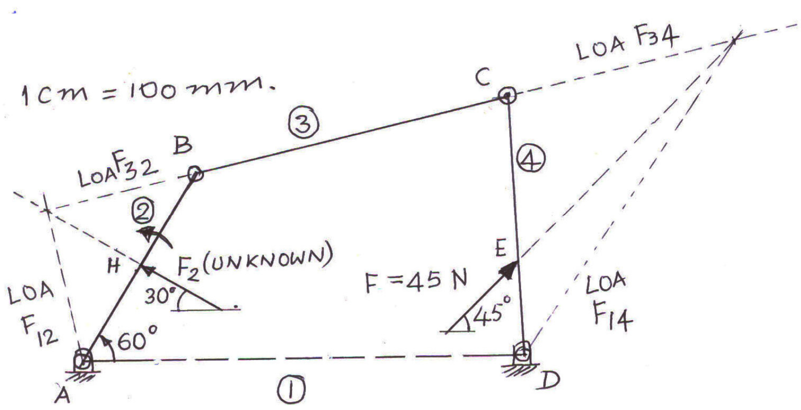
Friction axis: the new axis along which the thrust acts.

Problem No 8.

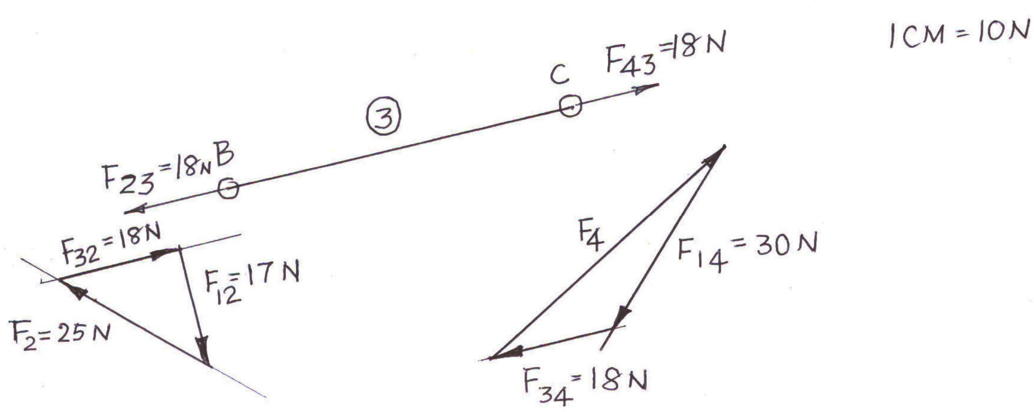
In a four bar mechanism ABCD, AB=350mm, BC=50mm, CD=400mm, AD=700mm, DE=150mm, $D\hat{A}B = 60^0$, AD is fixed. Determine the force on link AB required at the mid point, in the direction shown, for static equilibrium. μ=0.4 for each revolving pair. Assume CCW impending motion of AB. Radius of each journal is 50mm.

Also find the torque on AB for its impending CW motion.

Analysis for CCW motion



Solve the problem neglecting friction to know the magnitudes and directions of forces



Radius of the friction circle = μ x journal radius = 0.4 x 50 = 20 mm

Analysis with Friction considered---

AB rotates CCW, DC rotates CCW

∠*ABC* decreasing, LBCD increasing

At C:

∠BCD increases & 3 rotates CW w.r.t 4

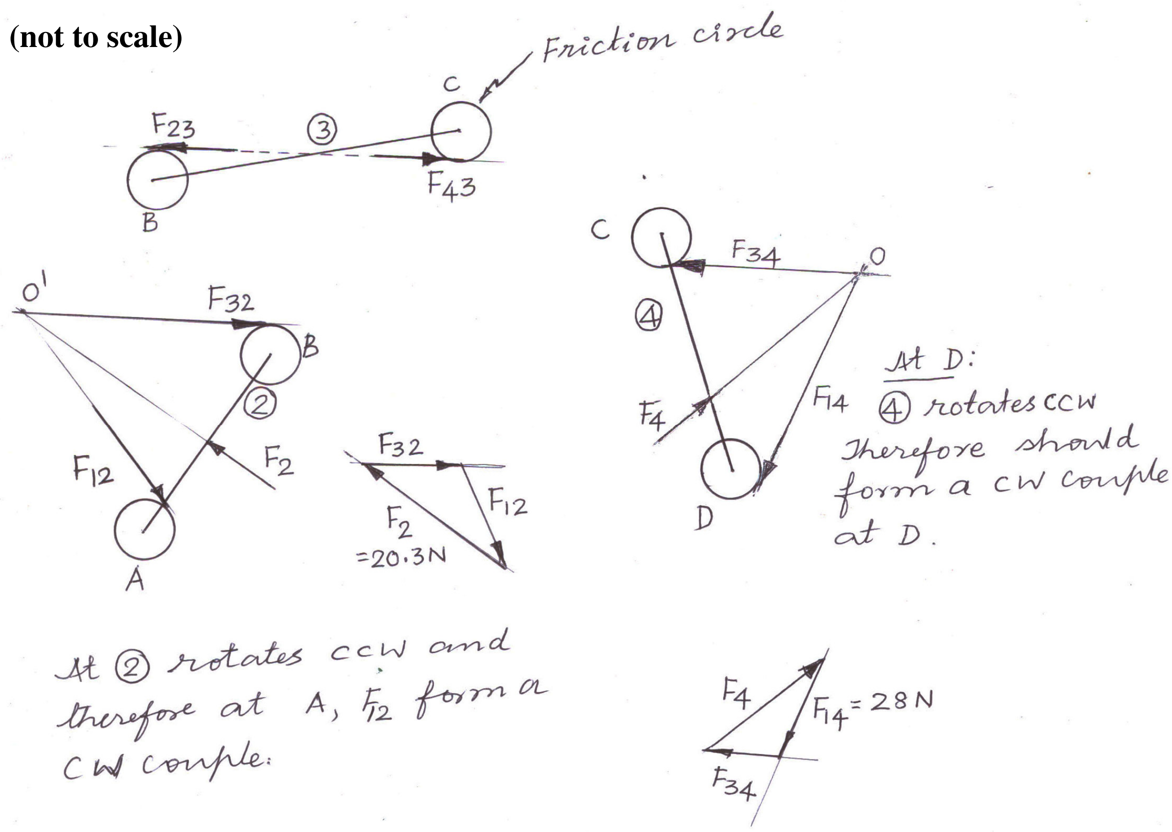
Therefore, F_{43} opposes the rotation of 4 by generating a CCW friction couple at C

At B:

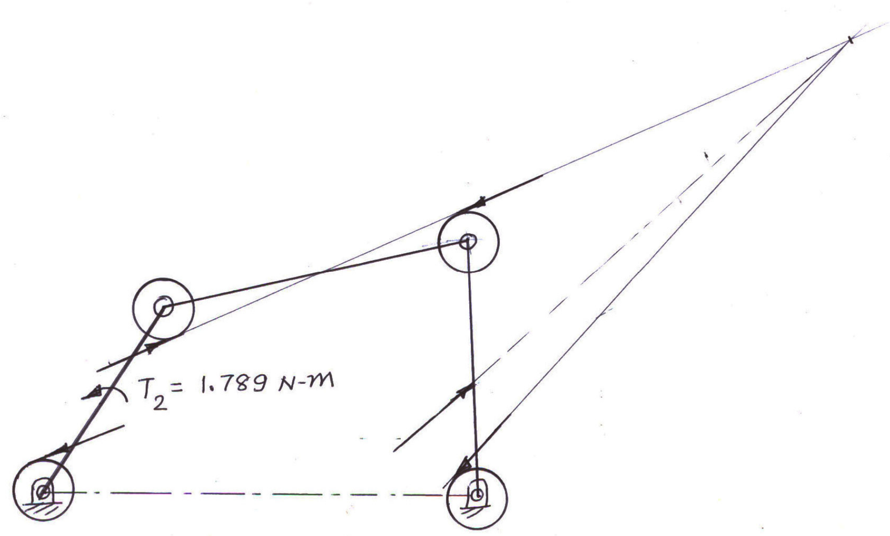
∠BCD decreases & 3 rotates CW w.r.t 2

Therefore, F_{23} forms a CCW friction couple at B

(not to scale)



For CW rotation of AB



References

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2) Mechanisms and Dynamics of machinery by Hamitton H.Mabie and Fred W.Ocvirk, John Wiley & sons, Newyork.
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4) Theory of Machines, by S.S Rathan, Tata McGraw-hill.
5) Mechanism & Machine Theory by Ashok G.Ambekar, Prentice Hall of India Pvt. Limited, New Delhi – 110001, 2007.

Summary

1) We have learnt how to analysis a mechanism for a given force system by graphical method.

2) It is also known to analysis a mechanism considering and not considering friction.

3) Now, the student should be able to analysis 4 bar, six bar and also engine mechanism for the given static force.