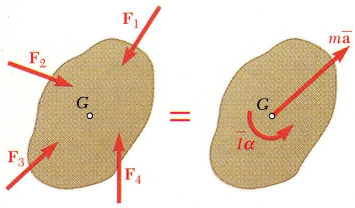
Engineering Mechanics

Dynamics and Vibrations

Engineering Mechanics – Dynamics & Vibrations

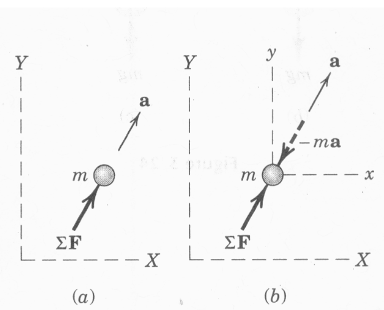
Plane Motion of a Rigid Body: Equations of Motion



- Motion of a rigid body in plane motion is completely defined by the resultant and moment resultant about the mass centre *G* of the external forces.

$$\sum F_x = m\bar{a}_x \quad \sum F_y = m\bar{a}_y \quad \sum M_G = J\alpha$$

D'Alembert's principle: inertia forces



- The particle acceleration we measure from a fixed set of axes X-Y-Z (Figure (a)) is its absolute acceleration ***a. In this case the familiar relation*** $\sum F = ma$ ***applies***
- When we observe the particle from a moving system x-y-z attached to the particle, the particle necessarily appears to be at rest or in equilibrium. A fictitious force ***-ma (so called inertia force)*** acts on the particle (figure b)

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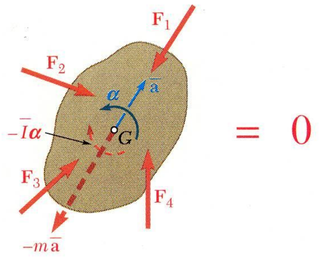
Plane Motion of a Rigid Body: **D'Alembert's principle**

D'Alembert's principle: inertia forces



- D'Alembert showed that one can transform an accelerating rigid body into an equivalent static system by adding the so-called "inertia forces"
  - The translational inertia must act through the center of mass and the rotational inertia can act anywhere. The system can then be analyzed exactly as a static system.
  - The inertia forces are seen to oppose the motion

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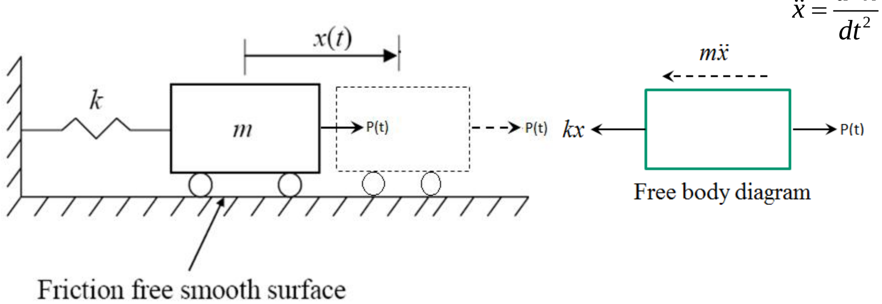
Spring-Mass System

Small Oscillations

$$\ddot{x} = \frac{d^2x}{dt^2}$$



Newton's second law

$$p(t) - kx = m\ddot{x}$$

D'Alembert's principle

$$\sum F_x = 0$$

$$m\ddot{x} + kx - p(t) = 0$$

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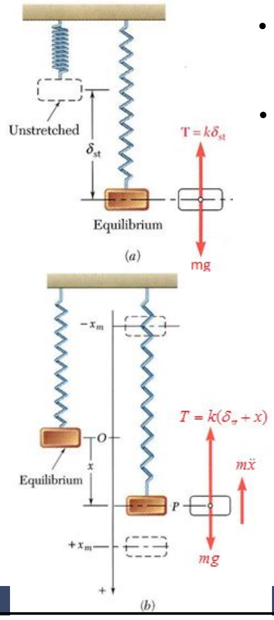
Spring-Mass System: Gravity Effect



- At static equilibrium configuration

$$k\delta_{st} = mg$$

- Now the particle is displaced through a distance x_o *from* its static equilibrium configuration and released with a velocity v_o, the particle will undergo *simple harmonic motion*

From the free body diagram of the mass *m* at a time instant t with displacement x(t)

$$\sum F_v = 0$$

$$mg - k\left(\delta_{st} + x\right) - m\ddot{x} = 0$$

$$m\ddot{x} + kx = 0 \Longrightarrow \text{Governing equation of motion}$$

$$\ddot{x} + \frac{k}{m}x = 0 \quad \text{where } \omega_n^2 = \frac{k}{m}$$

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Free Vibrations of Spring-Mass System

$$x(t) = C_1 \sin \omega_n t + C_2 \cos \omega_n t \qquad \text{General Solution}$$

- *x* is a *periodic function* and w_n is the ***natural circular frequency*** of motion.
- C_1 and C_2 are determined by the initial conditions:

$$x = C_1 \sin(\omega_n t) + C_2 \cos(\omega_n t) \qquad @\,time\ t=0;\ x=x_o \quad \Rightarrow C_2 = x_o$$

$$v = \dot{x} = C_1\omega_n \cos(\omega_n t) - C_2\omega_n \sin(\omega_n t) \qquad @\,time\ t=0;\ v=\dot{x}=v_o \quad \Rightarrow C_1 = \frac{v_o}{\omega_n}$$

$$x(t) = \frac{v_0}{\omega_n}\sin \omega_n t + x_o \cos \omega_n t$$

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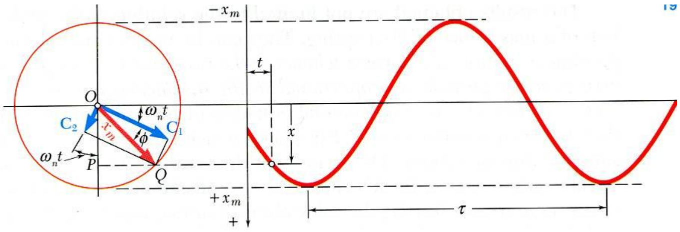
Simple Harmonic Motion

$\mathbf{C_1} = \dfrac{\mathbf{v_0}}{\boldsymbol{\omega_n}}$

$\mathbf{C_2} = \mathbf{x_0}$



- Displacement is equivalent to the x component of the sum of two vectors $\vec{C}_1 + \vec{C}_2$ which rotate with constant angular velocity ω_n.

$$x(t) = \frac{v_0}{\omega_n}\sin\omega_n t + x_o \cos\omega_n t$$

$$x = x_m \sin(\omega_n t + \phi)$$

$$x_m = \sqrt{(v_0/\omega_n)^2 + x_0^2} = \text{ amplitude}$$

$$\phi = tan^{-1}(v_0\omega_n/x_0) = \text{ phase angle}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \text{ period}$$

$$f_n = \frac{1}{\tau_n} = \frac{\omega_n}{2\pi} = \text{ natural frequency}$$

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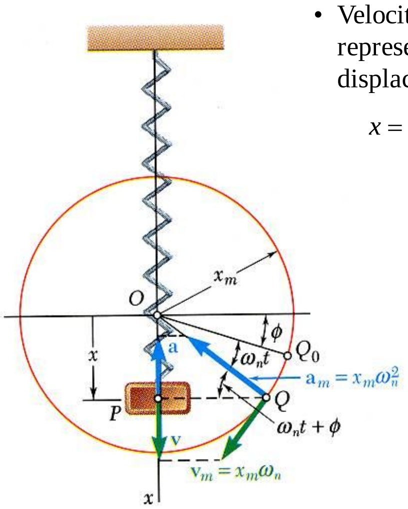
Simple Harmonic Motion



- Velocity-time and acceleration-time curves can be represented by sine curves of the same period as the displacement-time curve but different phase angles.

$$x = x_m \sin(\omega_n t + \phi)$$

$$\begin{aligned} v &= \dot{x} \\ &= x_m\omega_n \cos(\omega_n t + \phi) \\ &= x_m\omega_n \sin(\omega_n t + \phi + \pi/2) \end{aligned}$$

$$\begin{aligned} a &= \ddot{x} \\ &= -x_m\omega_n^2 \sin(\omega_n t + \phi) \\ &= x_m\omega_n^2 \sin(\omega_n t + \phi + \pi) \end{aligned}$$

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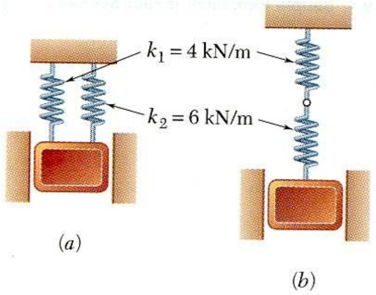
Sample Problem 1

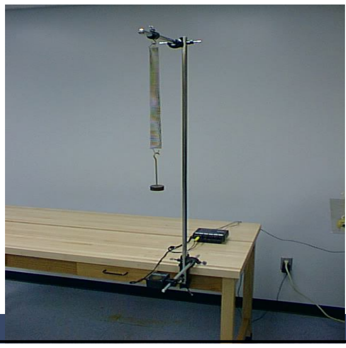


A 50-kg block moves between vertical guides as shown. The block is pulled 40mm down from its equilibrium position and released.

For each spring arrangement, determine *a*) the period of the vibration, *b*) the maximum velocity of the block, and *c*) the maximum acceleration of the block.

SOLUTION:

- For each spring arrangement, determine the spring constant for a single equivalent spring.
- Apply the D'Alembert's principle for the harmonic motion of a spring-mass system.



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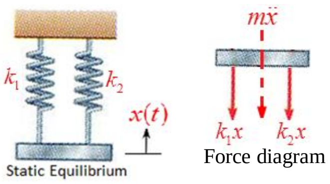
Sample Problem 1

$k_1 = 4\,\text{kN/m} \quad k_2 = 6\,\text{kN/m} \quad m = 50kg$



For equilibrium: $\sum F_v = 0$

$$m\ddot{x} + k_1x + k_2x = 0$$

$$m\ddot{x} + (k_1 + k_2)x = 0$$

$$m\ddot{x} + k_{eq}x = 0$$

$$k_{eq} = k_1 + k_2$$
$$= 10\,\text{kN/m} = 10^4\ \text{N/m}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{10^4}{50}} = 14.14\,\text{rad/s}\,ec$$

$$\tau_n = \frac{2\pi}{\omega_n}$$

$$\boxed{\tau_n = 0.444\,\text{s}}$$

$$v = \dot{x} = x_m \omega_n \cos(\omega_n t + \phi)$$

$$v_m = x_m \omega_n$$
$$= (0.040\,\text{m})(14.14\,\text{rad/s})$$

$$\boxed{v_m = 0.566\,\text{m/s}}$$

$$a = \ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi)$$

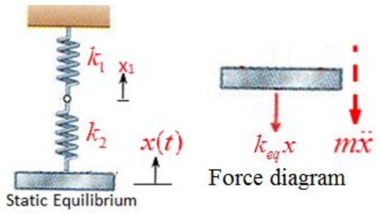
$$a_m = x_m \omega_n^2$$
$$= (0.040\ \text{m})(14.14\ \text{rad/s})^2$$

$$\boxed{a_m = 8.00\,\text{m/s}^2}$$

Engineering Mechanics – Dynamics & Vibrations

Sample Problem 1

$k_1 = 4\,\text{kN/m} \quad k_2 = 6\,\text{kN/m}$



$$P = k_{eq}x = k_2(x - x_1) = k_1 x_1$$

$$x_1 = \frac{k_2 x}{k_1 + k_2}$$

$$P = \frac{k_1 k_2}{k_1 + k_2} x$$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

For equilibrium: $\sum F_v = 0$

$$m\ddot{x} + k_{eq}x = 0$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2400\text{N/m}}{20\text{kg}}} = 6.93\text{rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n}$$

$\boxed{\tau_n = 0.907\text{ s}}$

$$v = \dot{x} = x_m \omega_n \cos(\omega_n t + \phi)$$

$$v_m = x_m \omega_n$$

$$= (0.040\text{ m})(6.93\text{ rad/s})$$

$\boxed{v_m = 0.277\text{ m/s}}$

$$a = \ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi)$$

$$a_m = x_m \omega_n^2$$

$$= (0.040\text{ m})(6.93\text{ rad/s})^2$$

$\boxed{a_m = 1.920\text{ m/s}^2}$

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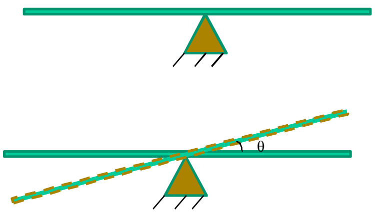
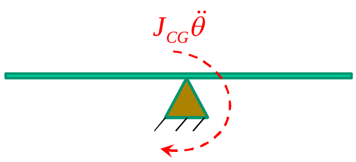
Distributed Mass: Rotational Inertia



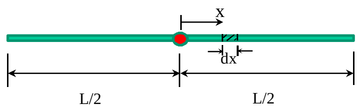
Displacement diagram



Force diagram

J = Mass moment of Inertia about C.G

$$J = \int r^2 dm$$



$$J = \int_{-L/2}^{L/2} x^2(\overline{m}dx)$$

$$\overline{m} = mass\,/\,unit\ length$$

$$= \frac{\overline{m}L^3}{12} = \frac{(\overline{m}L)L^2}{12} = \frac{ML^2}{12}$$

$$\boxed{J_{CG}\ddot{\theta} = \frac{mL^2}{12}\ddot{\theta}}$$

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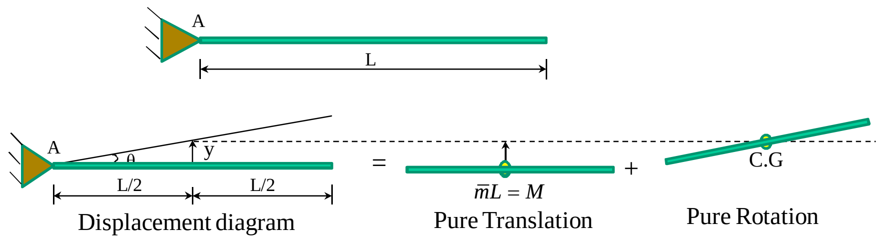
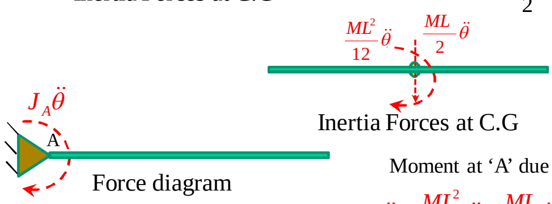
Distributed Mass: Rotational Inertia



Inertia Forces at C.G $= \; M\ddot{y} = \frac{ML}{2}\ddot{\theta} \; + \; J_{CG}\ddot{\theta}$



Moment at 'A' due to inertia forces

$$J_A\ddot{\theta} = \frac{ML^2}{12}\ddot{\theta} + \frac{ML}{2}\ddot{\theta}\left(\frac{L}{2}\right) = \frac{ML^2}{3}\ddot{\theta}$$

$$J_A = \frac{ML^2}{3}$$

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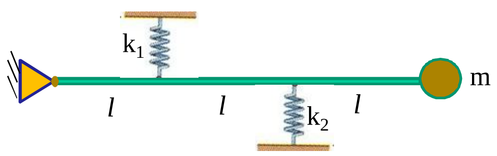
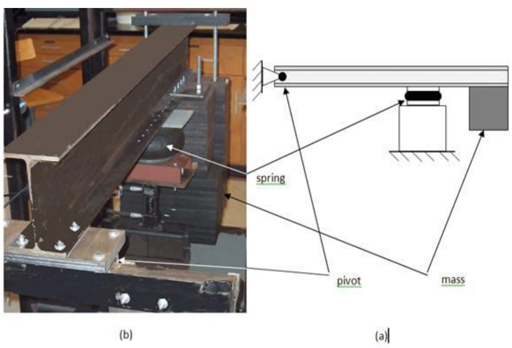
Sample Problem 2





For the system shown, m=0.4-kg, K_1=2N/mm and K_2=3N/mm. Taking the rod on which the mass is fixed as light and stiff.

Determine *a*) Natural frequency of the system, *b*) the period of the vibration.

<u>SOLUTION</u>:

- Select a degree of freedom (small displacement).
- Represent deformations of springs (for elastic forces) and masses (for inertia forces) in terms of x(t)

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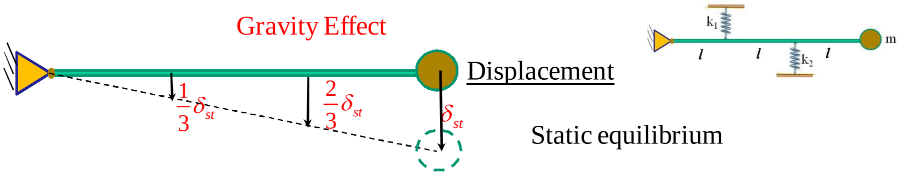
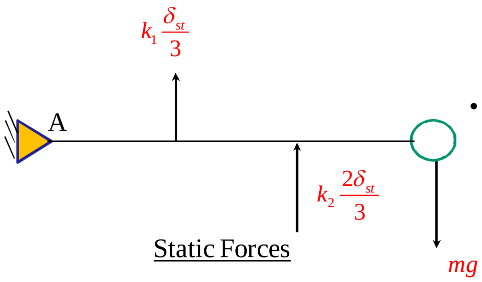
Sample Problem 2





- At static equilibrium configuration

$$\sum M_A = 0$$

$$k_1\frac{\delta_{st}}{9} + k_2\frac{4\delta_{st}}{9} = mg$$

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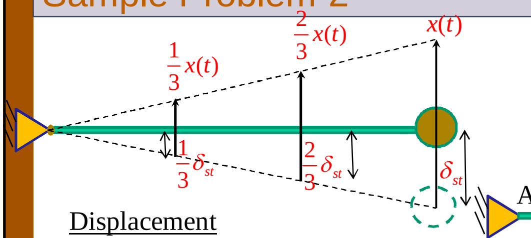
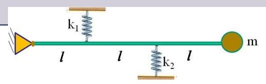
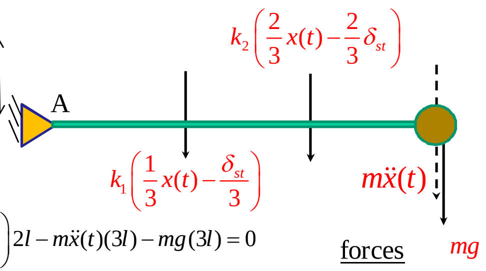
Sample Problem 2







For equilibrium: $\sum M_A = 0$

$$-k_1\left(\frac{1}{3}x(t) - \frac{\delta_{st}}{3}\right)l - k_2\left(\frac{2}{3}x(t) - \frac{2}{3}\delta_{st}\right)2l - m\ddot{x}(t)(3l) - mg(3l) = 0$$

$$-k_1\left(\frac{1}{3}x(t) - \frac{\delta_{st}}{3}\right)\frac{1}{3} - k_2\left(\frac{2}{3}x(t) - \frac{2}{3}\delta_{st}\right)\frac{2}{3} - m\ddot{x}(t) - mg = 0$$

$$-\left[m\ddot{x}(t) + \left(\frac{k_1}{9} + \frac{4k_2}{9}\right)x(t)\right] + \left[\left(\frac{k_1}{9} + \frac{4k_2}{9}\right)\delta_{st} - mg\right] = 0$$

$$\left[m\ddot{x}(t) + \left(\frac{k_1}{9} + \frac{4k_2}{9}\right)x(t)\right] = 0 \implies \text{Equation of Motion}$$

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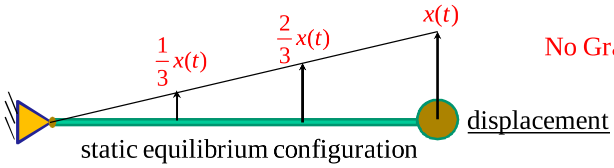
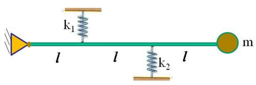
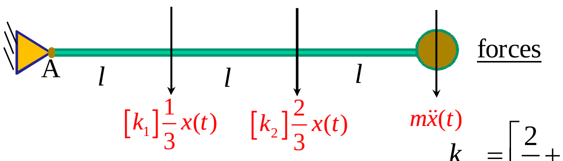
Sample Problem 2



No Gravity Effect





For equilibrium: $\sum M_A = 0$

$$\left[k_1\frac{1}{3}x(t)\right]l + \left[k_2\frac{2}{3}x(t)\right]2l + [m\ddot{x}(t)]3l = 0$$

$$m\ddot{x}(t) + \left[\frac{1}{3}k_1\frac{1}{3} + \frac{2}{3}k_2\frac{2}{3}\right]x(t) = 0$$

Equation of motion

$$m\ddot{x}(t) + k_{eq}x(t) = 0$$

$$k_{eq} = \left[\frac{2}{9} + \frac{3\text{x}4}{9}\right]\text{N/mm} = \frac{14}{9}\text{x}10^3\text{N/m}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{14000}{9\text{x}0.4}} = \boxed{62.36 \text{ rad/sec}}$$

$$\tau_n = \frac{2\pi}{\omega_n} \qquad \boxed{\tau_n = 0.100 \text{ sec}}$$

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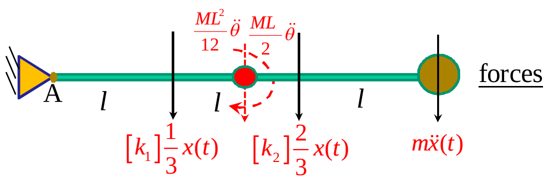
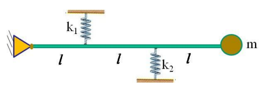
Sample Problem 2

If distributed mass of the bar is considered





For equilibrium: $\sum M_A = 0$

$$\left[k_1\frac{1}{3}x(t)\right]l + \left[k_2\frac{2}{3}x(t)\right]2l + [m\ddot{x}(t)]3l + \frac{ML^2}{3}\ddot{\theta} = 0$$

$$L = 3l$$

$$\ddot{\theta} = \frac{\ddot{x}(t)}{3l}$$

$$\left(m + \frac{M}{3}\right)\ddot{x}(t) + \left[k_1\frac{1}{9} + k_2\frac{4}{9}\right]x(t) = 0$$

one can also use principle of virtual work to obtain the equation of motion

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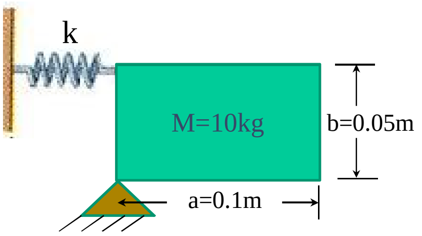
Sample Problem 3



SOLUTION:

- Select a degree of freedom (small displacement).
- Represent deformations of springs (for elastic forces) and masses (for inertia forces) in terms of θ

Derive the equation of motion of a rectangular block resting on a frictionless surface as shown for small oscillations in a horizontal plane . Solve the equation of motion by simplifying it for M=10Kg, a=0.1m, b=0.05m, k=10KN/m

Determine *a*)Natural frequency of the system, *b*) the period of the vibration.

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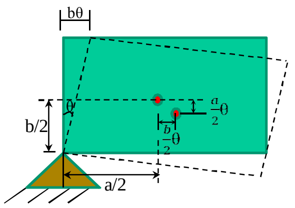
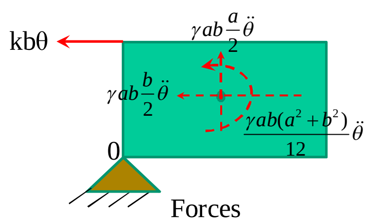
Sample Problem 3



Displacements



Forces

For equilibrium: $\sum M_0 = 0$ $\qquad$ $\gamma = mass / unit\ area$

$$\frac{\gamma ab(a^2+b^2)}{12}\ddot{\theta} + \gamma ab\frac{a}{2}\left(\frac{a}{2}\right)\ddot{\theta} + \gamma ab\frac{b}{2}\left(\frac{b}{2}\right)\ddot{\theta} + kb\theta(b) = 0 \qquad \gamma ab = M$$

$$\frac{M(a^2+b^2)}{12}\ddot{\theta} + M\frac{a^2}{4}\ddot{\theta} + M\frac{b^2}{4}\ddot{\theta} + kb^2\theta = 0$$

$$\left[\frac{m(a^2+b^2)}{3}\right]\ddot{\theta} + kb^2\theta = 0 \qquad J_o = \left[\frac{M(a^2+b^2)}{3}\right]$$

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Sample Problem 3

Equation of motion

$$0.0416\ddot{\theta} + 25\theta = 0$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{0.0416}} = \boxed{24.514 \text{ rad/sec}}$$

$$\tau_n = \frac{2\pi}{\omega_n} \qquad \boxed{\tau_n = 0.256 \text{ sec}}$$

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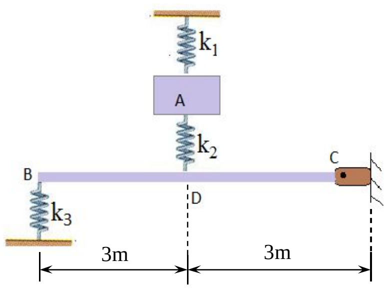
Sample Problem 4



SOLUTION:

- Select a degree of freedom (small displacement).
- Represent deformations of springs (for elastic forces) and masses (for inertia forces) in terms of x

What are the differential equation of motion about the static equilibrium configuration shown and the natural frequency of motion of body A for small motion of BC? Neglect inertia effects from BC. Assume K_1= 15 N/m, K_2= 20 N/m, K_3= 30 N/m and W_A=30N

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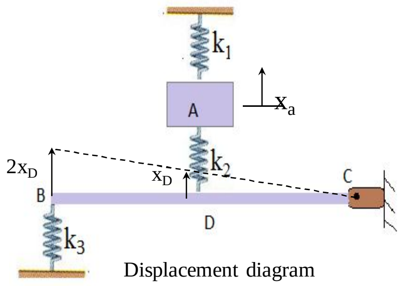
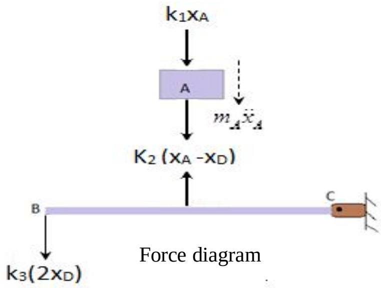
Sample Problem 4



Displacement diagram



Force diagram

The configuration shown is the static equilibrium and given that rod BC is mass less (i.e neglect the inertia effect of BC). Here two equilibrium conditions exist i.e

$\sum F_v = 0$ *and* $\sum M_c = 0$

$$\sum F_v = 0$$

$$m_A \ddot{x}_A + k_1 x_A + k_2 (x_A - x_D) = 0 \qquad (1)$$

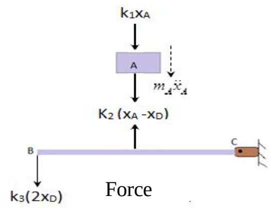
Engineering Mechanics – Dynamics & Vibrations

Sample Problem 4

$$\sum M_c = 0$$

$$-12k_3 x_D + 3k_2 (x_A - x_D) = 0 \qquad (2)$$

$$From \ (2), \quad x_D = \left(\frac{k_2}{k_2 + 4k_3} \right) x_A \qquad (3)$$



Force diagram

Thus due to inertia less rod BC the 2-dof problem reduces to 1-dof problem (since x_D depends purely on x_A).

Substitute (3) in (1)

$$m_A \ddot{x}_A + \left[k_1 + k_2 \left(\frac{4k_3}{k_2 + 4k_3} \right) \right] x_A = 0$$

$$\omega_n = \sqrt{\frac{k_1 k_2 + 4k_3 (k_1 + k_2)}{m_A (k_2 + 4k_3)}}$$

$$\boxed{\omega_n = 3.242 \text{ rad/sec}}$$

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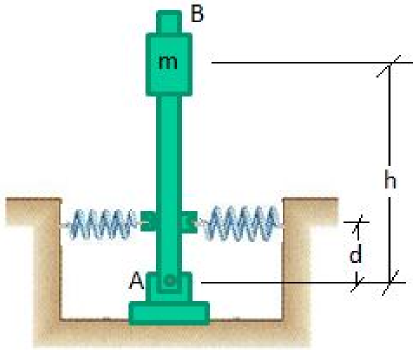
Sample Problem 5



Rod AB is attached to a hinge at A and to two springs, each of constant k. if h=700 mm, d=300 mm, and m=20 kg, determine the value of k for which the period of small oscillation is (a) 1sec, (b) infinite. Neglect the mass of the rod and assume that each spring can act in either tension or compression.

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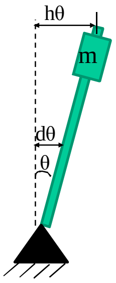
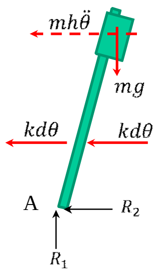
Sample Problem 5



Displacements



Forces

For equilibrium: $\sum M_A = 0$

$$mh\ddot{\theta}(h) + 2kd\theta(d) - mg(h\theta) = 0$$

$$\ddot{\theta} + \left[\frac{2kd^2}{mh^2} - \frac{g}{h}\right]\theta = 0$$

$$\omega_n^2 = \left[\frac{2kd^2}{mh^2} - \frac{g}{h}\right]$$

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Sample Problem 5

(a) For $\tau = 1\,\text{sec}$, $\tau = \frac{2\pi}{\omega_n}$, $\omega_n^2 = 4\pi^2$

$$4\pi^2 = \frac{2k}{20}\left(\frac{0.3}{0.7}\right)^2 - \frac{9.81}{0.7}$$

k = 2912.4 N/m

(b) For τ = infinite, $\tau = \frac{2\pi}{\omega_n}$, $\omega_n = 0$

$$\omega_n^2 = 0 = \frac{2k}{20}\left(\frac{0.3}{0.7}\right)^2 - \frac{9.81}{0.7}$$

k = 763.0 N/m

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Sample Problem 5

One can also use principle of minimum potential energy to obtain k when T is infinite

ω_n=0 which means the rod have $T_n \rightarrow \infty$,i.e.

For small disturbance θ, it will never return to original position (θ=**0**)

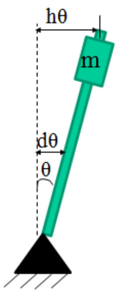
By minimum potential energy, We have

$$V = mg(h\cos\theta) + 2\left[\frac{1}{2}k(d\theta)^2\right]$$

For equilibrium

$$\frac{dV}{d\theta} = -mg(h\sin\theta) + kd^2(2\theta) = 0$$

$\theta=0^0$ is a equilibrium configuration



Displacements

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Sample Problem 5

For stability of Equilibrium: $\left.\dfrac{d^2V}{d\theta^2}\right|_{\theta=0} > 0$

$$\frac{d^2V}{d\theta^2} = -mg(h\cos\theta) + 2kd^2$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0} = -mgh + 2kd^2 > 0$$

$$k > \frac{mgh}{2d^2}$$

k >763 N/m

i.e. $\theta=0^0$,Configuration is stable for k >763N/m (which is same as that of vibrational analysis)

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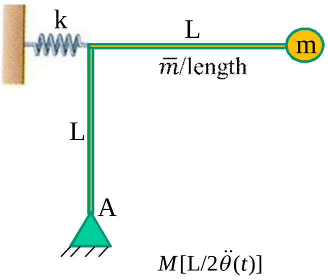
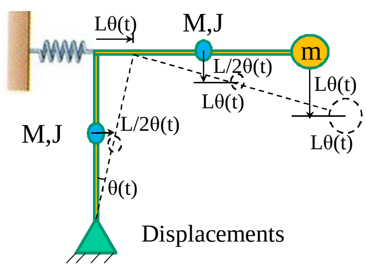
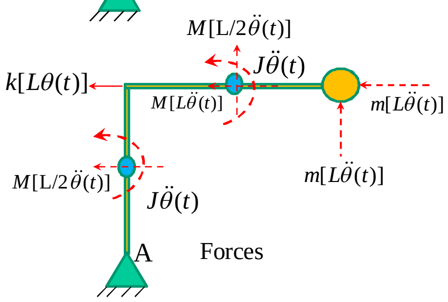
Sample Problem 6

$M = \bar{m}L$

$J = \dfrac{ML^2}{12}$







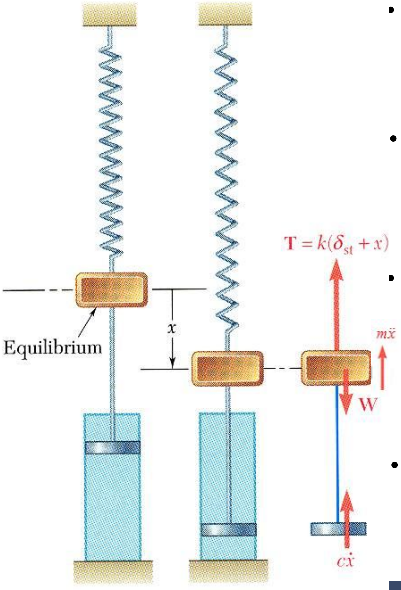
For equilibrium: $\sum M_A = 0$

$$mL^2\ddot{\theta} + mL^2\ddot{\theta} + \frac{ML^2}{4}\ddot{\theta} + ML^2\ddot{\theta} + \frac{ML^2}{12}\ddot{\theta} + \frac{ML^2}{12}\ddot{\theta} + \frac{ML^2}{4}\ddot{\theta} - kL^2\theta = 0$$

$$\left(2mL^2 + \frac{5}{3}ML^2\right)\ddot{\theta} - kL^2\theta = 0$$

Engineering Mechanics – Dynamics & Vibrations

Damped Free Vibrations



- All vibrations are damped to some degree by forces due to *dry friction*, *fluid friction*, or *internal friction*.

- With *viscous damping* due to fluid friction,

  $+\downarrow \sum F = ma: \quad W - k(\delta_{st} + x) - c\dot{x} = m\ddot{x}$

  $m\ddot{x} + c\dot{x} + kx = 0$

- Substituting $x = e^{\lambda t}$ and dividing through by $e^{\lambda t}$ yields the *characteristic equation*,

  $m\lambda^2 + c\lambda + k = 0 \qquad \lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$

- Define the critical damping coefficient such that

  $\left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} = 0 \qquad c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$

Engineering Mechanics – Dynamics & Vibrations

Damped Free Vibrations

- Characteristic equation,

  $m\lambda^2 + c\lambda + k = 0 \qquad \lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$

  $c_c = 2m\omega_n = \textit{critical damping coefficient}$

Define damping ratio

$\xi = \frac{c}{c_c} = \frac{c}{2m\omega_n}$

$\lambda = -\xi\omega_n \pm i\omega_n\sqrt{1-\xi^2}$

- *Light damping*: $c < c_c$

  $x = e^{-(c/2m)t}\left(C_1 \sin\omega_d t + C_2 \cos\omega_d t\right)$

  $= e^{-\xi\omega_n t}\left(C_1 \sin\omega_d t + C_2 \cos\omega_d t\right)$

  $\omega_d = \omega_n\sqrt{1-\xi^2} =$ damped frequency

  Underdamped System

- *Critical damping*: $c = c_c$

  $x = (C_1 + C_2 t)e^{-\omega_n t}$
  - double roots
  - nonvibratory motion

  Critically damped System

- *Heavy damping*: $c > c_c$

  $x = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$
  - negative roots
  - nonvibratory motion

  Overdamped System

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Damped Vs. Undamped Free Vibrations

Damped

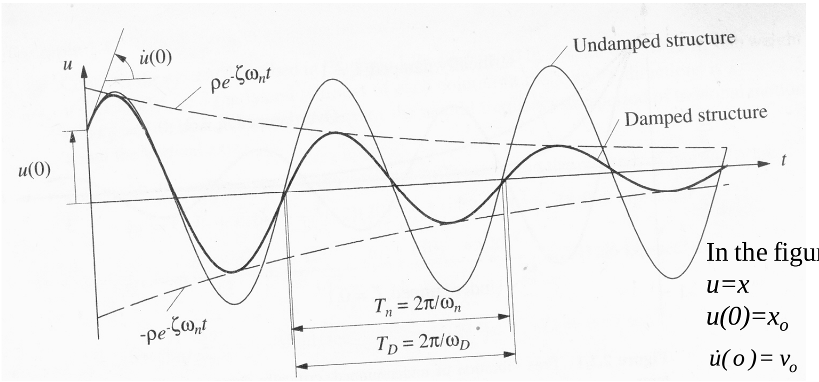
$$x(t) = e^{-\xi\omega_n t}\left[x_o \cos\omega_D t + \frac{v_o + \xi\omega_n x_o}{\omega_D}\sin\omega_D t\right]$$

$$\rho = \sqrt{x_o^2 + \left[\frac{v_o + \xi\omega_n x_o}{\omega_D}\right]^2}$$

Undamped

$$x = x_o \cos(\omega_n t) + \frac{v_o}{\omega_n}\sin(\omega_n t)$$

$$\rho = \sqrt{x_o^2 + \left[\frac{v_o}{\omega_n}\right]^2}$$



In the figure use:
$u = x$
$u(0) = x_o$
$\dot{u}(o) = v_o$

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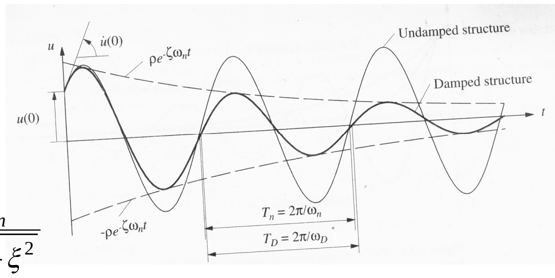
Damped Free Vibrations (logarithmic decrement)

From the two successive peaks

$$\frac{x_n}{x_{n+1}} = e^{\xi\omega_n T_D}$$



$$\omega_d = \omega_n\sqrt{1-\xi^2} \Rightarrow T_D = \frac{T_n}{\sqrt{1-\xi^2}}$$

$$Note, T_n = \frac{2\pi}{\omega_n} \; and \; \xi = \frac{c}{2m\omega_n}$$

$$\ln\frac{x_n}{x_{n+1}} = \xi\omega_n T_D = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

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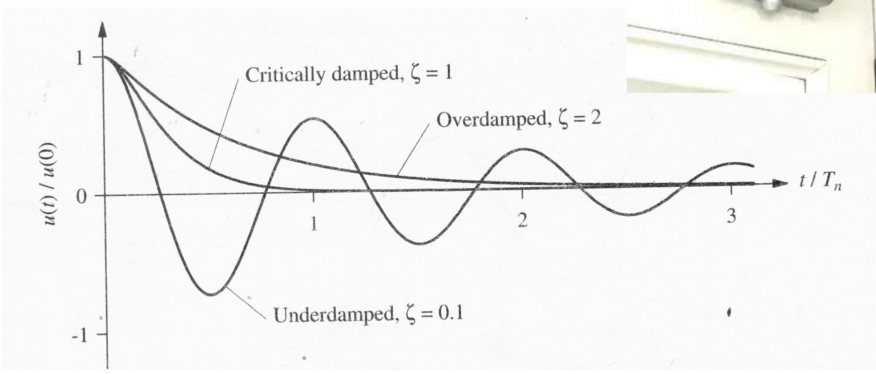
Damped Free Vibrations



$$For \quad \dot{u}(o) = v_o = 0 , i.e \, no \, initial \, velocity$$

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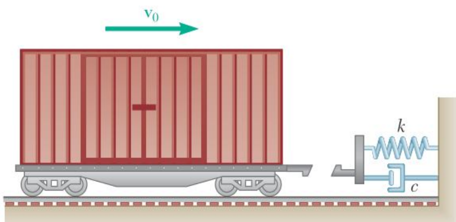
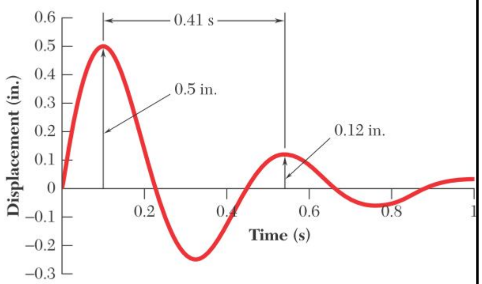
Sample Problem





A loaded railroad car weighing 30,000 lb is rolling at a constant velocity v_o when it couples with a spring and dashpot bumper system. The recorded displacement-time curve of the loaded railroad car after coupling is as shown. Determine (a) the damping constant, (b) the spring constant.

Source: BJ

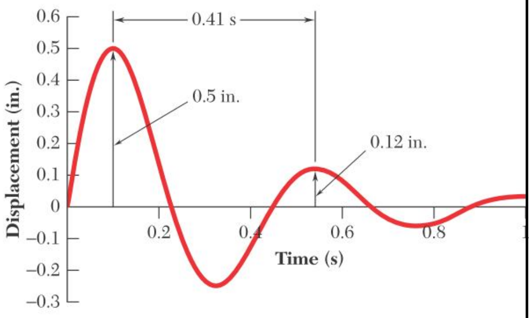
Engineering Mechanics – Dynamics & Vibrations

Sample Problem

$$\tau_d = 0.41 \text{ s}$$

$$\omega_d = \frac{2\pi}{\tau_d} = \frac{2\pi}{0.41} = 15.325 \text{ rad/s}$$

$$\frac{x_1}{x_2} = e^{\xi\omega_n\tau_D} = e^{\frac{c}{2m_n}\tau_D}$$



$$\frac{c\tau d}{2m} = \ln\left(\frac{x_1}{x_2}\right)$$

$$c = \frac{2m}{\tau_d}\ln\frac{x_1}{x_2}$$

$$= \frac{(2)(15000)}{0.41}\ln\frac{12.5}{3}$$

$$= 104.423\times10^3 \text{ N-s/m}$$

$$\omega_d^2 = \frac{k}{m} - \left(\frac{c}{2m}\right)^2$$

$$k = m\omega_d^2 + \frac{c^2}{4m}$$

$$= (15000)(15.325)^2 + \frac{(104.423\times10^3)^2}{(4)(15000)}$$

$$= 3.7046\times10^6 \text{ N/m}$$