Chapter - 2

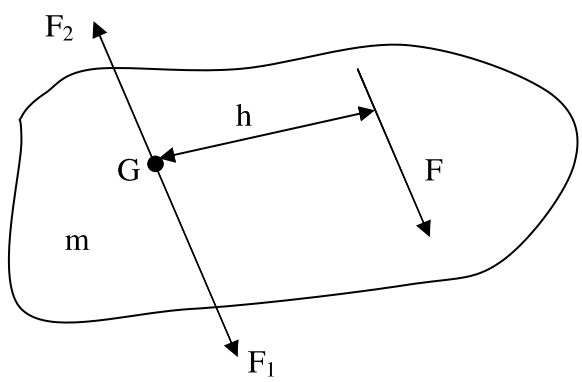
DYNAMIC FORCE ANALYSIS:

It is defined as the study of the force at the pin and guiding surfaces and the forces causing stresses in machine parts, such forces being the result of forces due to the motion of each part in the machine. The forces include both external and inertia forces. Inertia forces in high speed machines become very large and cannot be neglected, Ex: Inertia force of the piston of an automobile travelling at high speed might be thousand times the weight of the piston. The dynamic forces are associated with accelerating masses.

If each link, with its inertia force and force applied to the link can be considered to be in equilibrium, the entire system can also be considered to be in equilibrium.

Determination of force & couple of a link

(Resultant effect of a system of forces acting on a rigid body)



G = c .g point
F_1& F_2: equal and opposite forces acting through G (Parallel to F)

F: Resultant of all the forces acting on the rigid body.
h: perpendicular distance between F & G.
m = mass of the rigid body

Note: F_1=F_2 & opposite in direction; they can be cancelled with out affecting the equilibrium of the link. Thus, a single force 'F' whose line of action is not through G, is capable of producing both linear & angular acceleration of CG of link.

F and F_2 form a couple.

$T = F \times h = I\,\alpha = mk^2\,\alpha$ (Causes angular acceleration) . . . . . . (1)

Also, F_1 produces linear acceleration, f.

$F_1 = mf$

Using 1 & 2, the values of 'f' and 'α' can be found out if F_1, m, k & h are known.

D'Alembert's principle:

Final design takes into consideration the combined effect of both static and dynamic force systems. D'Alembert's principle provides a method of converting dynamics problem into a static problem.

Statement:

The vector sum of all external forces and inertia forces acting upon a rigid body is zero. The vector sum of all external moments and the inertia torque, acting upon the rigid body is also separately zero.

In short, sum of forces in any direction and sum of their moments about any point must be zero.

Inertia force and couple:

Inertia: Tendency to resist change either from state of rest or of uniform motion

Let 'R' be the resultant of all the external forces acting on the body, then this 'R' will be equal to the product of mass of the body and the linear acceleration of c.g of body. The force opposing this 'R' is the inertia force (equal in magnitude and opposite in direction).

(Inertia force is an Imaginary force equal and opposite force causing acceleration)

If the body opposes angular acceleration (α) in addition to inertia force R, at its cg, there exists an inertia couple Ig x α, Where Ig= M I about cg. The sense of this couple opposes α. i.e., inertia force and inertia couple are equal in magnitude to accelerating force and couple respectively but, they act in opposite direction.

Inertia force (F_i) = M x f,
(mass of the rigid body x linear acceleration of cg of body)

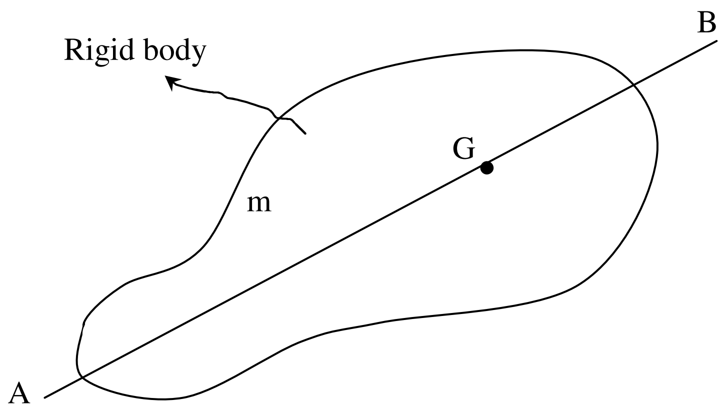
Inertia couple (C_i)=I x α, [MMI of the rigid body about an axis perpendicular to the plane of motion] [Angular acceleration]

Dynamic Equivalence:

The line of action of the accelerating force can also be determined by replacing the given link by a dynamically equivalent system. Two systems are said to be dynamically equivalent to one another, if by application of equal forces, equal linear and angular accelerations are produced in the two systems.

i.e., the following conditions must be satisfied;

i) The masses of the two systems must be same.
ii) The cg's of the two systems must coinside.
iii) The moments of inertia of the two systems about same point must be equal, Ex: about an axis through cg.

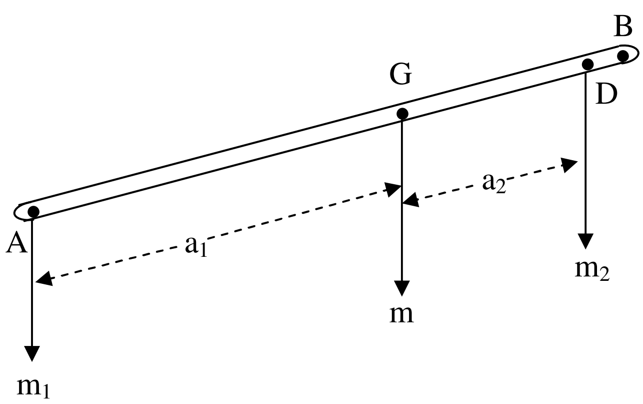


G = c.g.

m = mass of the rigid body

k_g = radius of gyration about an axis through G and perpendicular to the plane

Now, it is to be replaced by dynamically equivalent system.



m_1, m_2 – masses of dynamically equivalent system at a_1 & a_2 from G (respectively)

As per the conditions of dynamic equivalence,

$$m = m_1 + m_2 \quad \text{.. (a)}$$
$$m_1 a_1 = m_2 a_2 \quad \text{.. (b)}$$
$$mk_g^2 = m_1 a_1^2 + m_2 a_2^2 \quad \text{.. (c)}$$

Substituting (b) in (c),

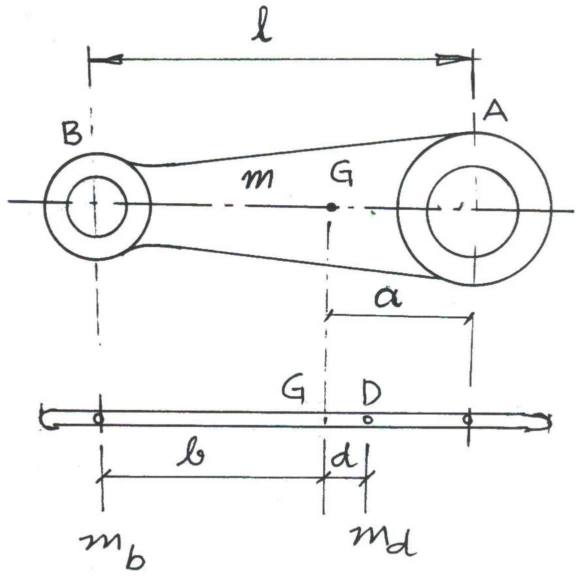
$$mk_g^2 = (m_2 a_2) a_1 + (m_1 a_1) a_2$$

$$= a_1 a_2 (m_2+m_1) = a_1 a_2 (m)$$

i.e., $k_g^2 = a_1 a_2$ $\qquad [I_g = mk_g^2 \ or \ k_g^2 = \frac{I_g}{m}]$

or $\frac{I_g}{m} = a_1 \, a_2$

Inertia of the connecting rod:



Connecting rod to be replaced by a massless link with two point masses m_b & m_d .

m = Total mass of the CR m_b& m_d point masses at B& D.

$$m_b + m_d = m \qquad -- \quad (i)$$

$$m_b \times b = m_d \times d \qquad -- \quad (ii)$$

Substituting (ii) in (i);

$$m_b + \left(m_b \times \frac{b}{d} \right) = m$$

$$m_b \left(1 + \frac{b}{d} \right) = m \qquad \text{or} \quad m_b \left(\frac{b+d}{d} \right) = m$$

$$\text{or } m_b = m \left(\frac{d}{b+d} \right) \qquad -- \quad (1)$$

Similarly; $$m_d = m \left(\frac{b}{b+d} \right) \qquad -- \quad (2)$$

Also; $I = m_b b^2 + m_d d^2$

$$= m \left(\frac{d}{b+d} \right) b^2 + m \left(\frac{b}{b+d} \right) d^2 \qquad [from\ (1) \& (2)]$$

$$I = mbd \left(\frac{b+d}{b+d} \right) = mbd$$

Then, $mk_g^2 = mbd,$ (since $I = mk_g^2$)

$$k_g^2 = bd$$

The result will be more useful if the 2 masses are located at the centers of bearings A & B.

Let m_a = mass at A and dist. AG = a

Then,

$$m_a + m_b = m$$

$$m_a = m\left(\frac{b}{a+b}\right) = m\frac{b}{l} \;; \quad Since\ (a+b=l)$$

Similarly, $$m_b = m\left(\frac{a}{a+b}\right) = m\frac{a}{l} \;; \quad (Since,\ a+b=l)$$

$$I^1 = m_a^{\,a^2} + m_b^{\,b^2} = \ldots = mbd$$

(Proceeding on similar lines it can be proved)

Assuming; $a > d,\ I^1 > I$

i.e., by considering the 2 masses A & B instead of D and B, the inertia couple (torque) is increased from the actual value. i.e., there exists an error, which is corrected by applying a correction couple (opposite to the direction of applied inertia torque).

The correction couple,

$$\Delta T = \alpha_c (mab - mbd)$$

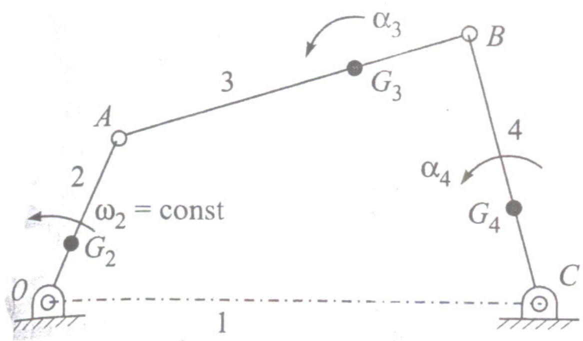
$$= mb\ \alpha_c\ (a-d)$$

$$= mb\ \alpha_c \left[(a+b) - (b+d)\right]$$

$$= mb\ \alpha_c\ (l-L) \qquad because\,(b+d=L)$$

As the direction of applied inertia torque is always opposite to the direction of angular acceleration, the direction of the correction couple will be same as that of angular acceleration i.e., in the direction of decreasing angle β.

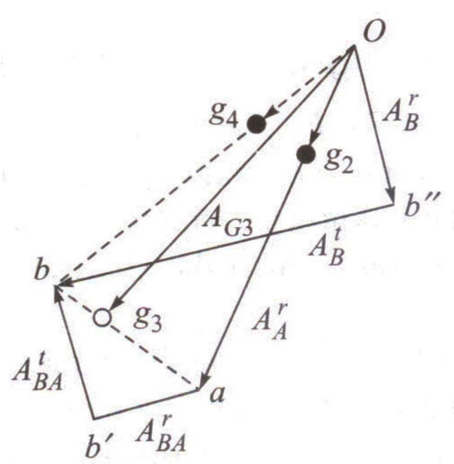
Dynamic force Analysis of a 4 – link mechanism.



OABC is a 4–bar mechanism. Link 2 rotates with constant ω_2. G_2, G_3 & G_4 are the cgs and M_1, M_2 & M_3 the masses of links 1, 2 & 3 respectively.

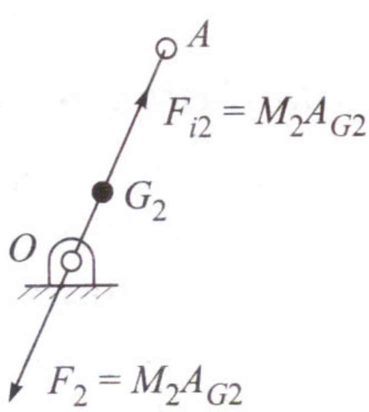
What is the torque required, which, the shaft at o must exert on link 2 to give the desired motion?

1. Draw the velocity & acceleration polygons for determing the linear acceleration of G_2, G_3 & G_4.
2. Magnitude and sense of α_3 & α_4 (angular acceleration) are determined using the results of step 1.



To determine inertia forces and couples

Link 2

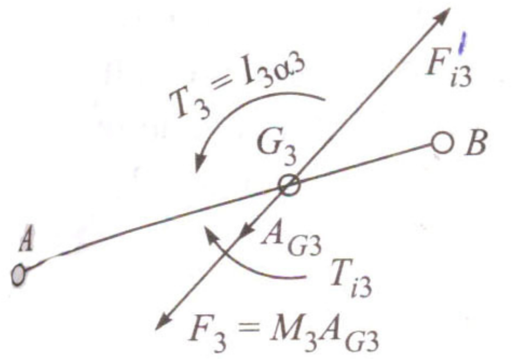


F_2 = accelerating force (towards O)

F_{i2} = inertia force (away from O)

Since ω_2 is constant, $\alpha_2 = 0$ and no inertia torque involved.

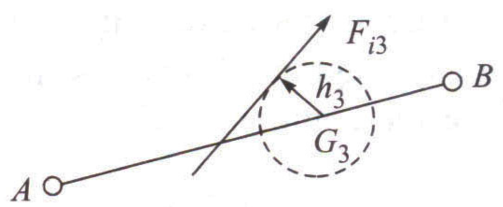
Link 3



Linear acceleration of G_3 (i.e., AG_3) is in the direction of Og_3 of acceleration polygon.

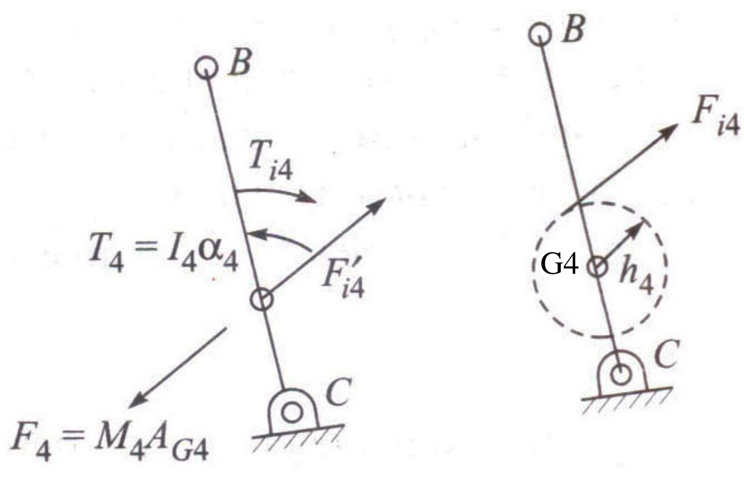
F_3 = accelerating force

Inertia force $F_{i3}^{'}$ acts in opposite direction. Due to α_3, there must be a resultant torque $T_3 = I_3\ \alpha_3$ acting in the sense of α_3 (I_3 is MMI of the link about an axis through G_3, perpendicular to the plane of paper). The inertia torque T_{i3} is equal and opposite to T_3.



F_{i3} can replace the inertia force $F_{i3}^{'}$ and inertia torque T_{i3}. F_{i3} is tangent to circle of radius h_3 from G_3, on the top side of it so as to oppose the angular acceleration α_3. $h3 = \frac{I_3\alpha_3}{M_3 AG_3}$

Link 4



$$h4 = \frac{I_4\alpha_4}{M_4 AG_4}$$

Problem 1 :

It is required to carryout dynamic force analysis of the four bar mechanism shown in the figure.

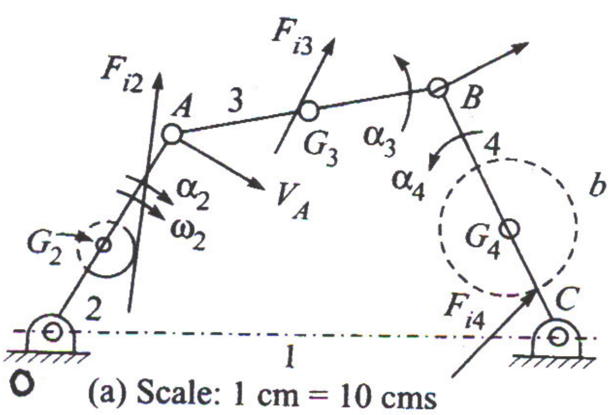
ω_2 = 20rad /s (cw), α_2 = 160 rad/s^2 (cw)

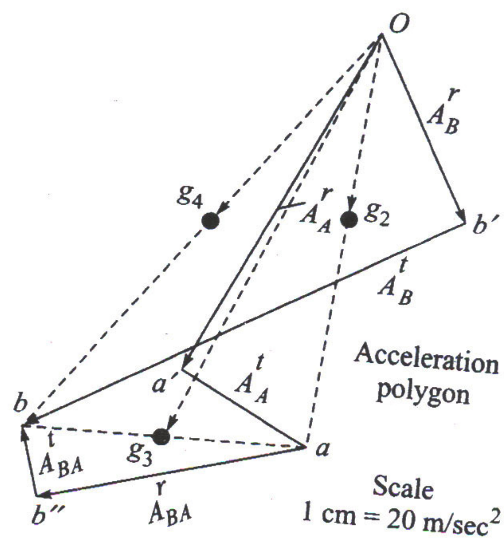
OA= 250mm, OG_2= 110mm, AB=300mm, AG_3=150mm, BC=300mm, CG_4=140mm, OC=550mm, $\angle AOC = 60°$

The masses & MMI of the various members are

Link	Mass, m	MMI (I_G, Kgm^2)
2	20.7kg	0.01872
3	9.66kg	0.01105
4	23.47kg	0.0277

Determine i) the inertia forces of the moving members
ii) Torque which must be applied to (2)





A) Inertia forces:

(i) (from velocity & acceleration analysis)

$V_A = 250 \times 20;\ 5 m/s, \quad V_B = 4\ m/s, \quad V_{BA} = 4.75\ m/s$

$a_A^r = 250 \times 20^2;\ 100 m/s^2, \quad a_A^t = 250 \times 160;\ 40 m/s^2$

Therefore;

$$A_B^r = \frac{V_B^2}{CB} = \frac{(4)^2}{0.3} = 53.33\ m/s^2$$

$$A_{BA}^r = \frac{V_{BA}^2}{B_A} = \frac{(4.75)^2}{0.3} = 75.21\ m/s^2$$

$$Og_2 = A_{G2} = 48\ m/s^2\ ;\ Og_3 = AG_3 = 120 m/s^2$$

$$Og_4 = A_{G4} = 65.4\ m/s^2$$

$$\alpha_3 = \frac{A_{BA}^t}{AB} = \frac{19}{0.3} = 63.3\ rad/s^2$$

$$\alpha_4 = \frac{A_B^t}{CB} = \frac{129}{0.3} = 430\ rad/s^2$$

Inertia forces (accelerating forces)

$$F_{G2} = m_2\ A_{G2} = \frac{20.7}{9.81} \times 48 = 993.6N\ (in\ the direction\ of\ Og_2)$$

$$F_{G3} = m_3\ A_{G3} = 9.66 \times 120 = 1159.2\,N\ (in\ the\ direction\ of\ Og_3)$$

$$= F_{G4} = m_4\ A_{G4} = 23.47 \times 65.4 = 1534.94N\ (in\ the direction of\ Og_4)$$

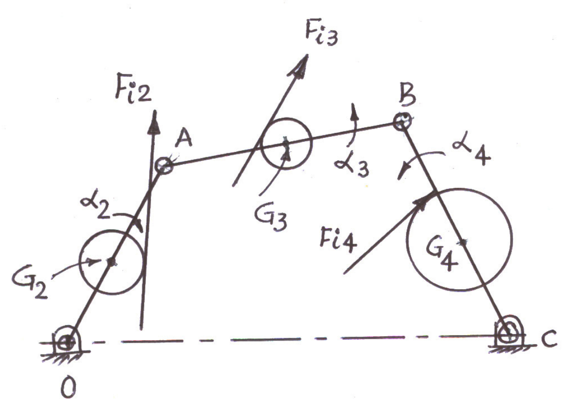
$$h_2 = \frac{I_{G2}(\alpha_2)}{F_2} = \frac{(0.01872 \times 160)}{993.6} = 3.01 \times 10^{-3} m$$

$$h_3 = \frac{I_{G3}(\alpha_3)}{F_3} = \frac{(0.01105 \times 63.3)}{1159.2} = 6.03 \times 10^{-4} m$$

$$h_4 = \frac{I_{G4}(\alpha_4)}{F_4} = \frac{(0.0277 \times 430)}{1534.94} = 7.76 \times 10^{-3} m$$

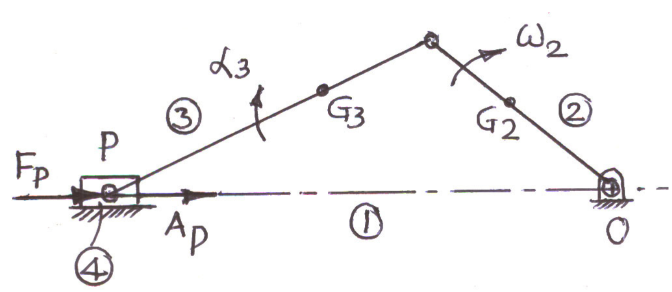
The inertia force F_{i2}, F_{i3} & F_{i4} have magnitudes equal and direction opposite to the respective accelerating forces and will be tangents to the circles of radius h_2, h_3 & h_4 from G_2, G_3 & G_4 so as to oppose α_2, α_3 & α_4.

$F_{i2} = 993.6N$, $F_{i3} = 1159.2N$ $F_{i4} = 1534.94N$



Further, each of the links is analysed for static equilibrium under the action of all external force on that link plus the inertia force.

<u>Dynamic force analysis of a slider crank mechanism.</u>



F_p = load on the piston

Link	mass	MMI
2	m_2	I_2
3	m_3	I_3
4	m_4	-

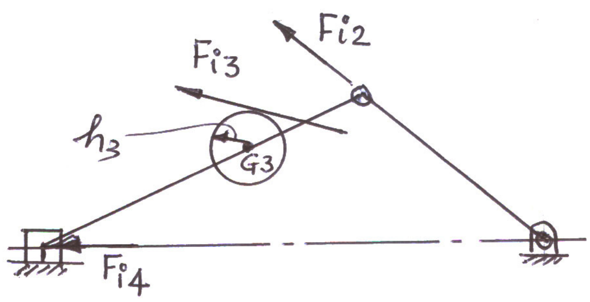
ω_2 assumed to be constant

Steps involved:

1. Draw velocity & acceleration diagrams
2. Consider links 3 & 4 together and single FBD written (elimination F_{34} & F_{43})
3. Since, weights of links are smaller compared to inertia forces, they are neglected unless specified.
4. Accelerating forces F_2 , F_3 & F_4 act in the directions of respective acceleration vectors Og_2, Og_3 & Og_p

Magnitudes: $F_2 = m_2\, AG_2 \quad F_3 = m_3\, AG_3 \quad F_4 = m_4\, A_p$

$F_{i2} = F_2$, $F_{i3} = F_3$, $F_{i4} = F_4$ (Opposite in direction)



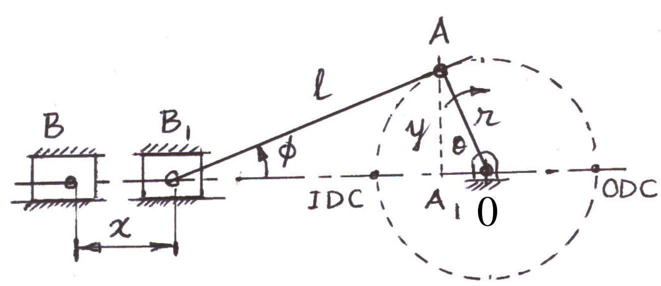
$$h_3 = \frac{I_3 \alpha_3}{M_3 \alpha_{g3}}$$

F_{i3} is tangent to the circle with h_3 radius on the RHS to oppose α_3

Solve for T_2 by solving the configuration for both static & inertia forces.

Dynamic Analysis of slider crank mechanism (Analytical approach)

Displacement of piston



x = displacement from IDC

$$x = BB_1 = BO - B_1O$$
$$= BO - (B_1A_1 + A_1O)$$
$$= (l+r) - (l\cos\phi + r\cos\theta) \qquad \left(\text{since}, \frac{l}{r} = n\right)$$
$$= (nr + r) - (rn\cos\phi + r\cos\theta)$$
$$= r[(n+1) - (n\cos\phi + \cos\theta)] \qquad \cos\phi = \sqrt{1-\sin^2\phi}$$

$$= r\left[(n+1) - (\sqrt{n^2 - \sin^2\theta} + \cos\theta)\right]$$ $$= \sqrt{1 - \frac{y^2}{l^2}}$$

$$= r\left[(1-\cos\theta) + (n - \sqrt{n^2 - \sin^2\theta})\right]$$ $$= \sqrt{1 - \frac{(r\sin\theta)^2}{l^2}}$$

$(similary\ \ l >> r, \frac{l}{r} = n >> 1$ & max $value\, of$ $\sin\ \theta = 1)$ $$= \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$\therefore \sqrt{n^2 - \sin^2\theta} \rightarrow \sqrt{n^2}\ or\ n),$

$$\boxed{x = r\ (1-\cos\theta)}$$ $$= \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

This represents SHM and therefore Piston executes SHM.

Velocity of Piston:

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\ \frac{d\theta}{dt}$$

$$\frac{d}{d\theta}\left[r(1-\cos\theta) + n - (n^2 - \sin 2\theta)^{-\frac{1}{2}}\right]\frac{d\theta}{dt}$$

$$= r\left[0 + \sin\theta + 0 - \frac{1}{2}(n^2 - \sin 2\theta)^{-1/2}(-2\sin\theta\cos\theta)\right]\omega$$

$$= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

Since, $n^2 >> \sin^2\theta$,

$$\therefore\ \ v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

Since n is quite large, $\frac{\sin 2\theta}{2n}$ can be neglected.

$$\boxed{\therefore\ \ v = r\omega\ \sin\theta}$$

Acceleration of piston:

$$a = \frac{dv}{dt} = \frac{dv}{d\theta}\ \frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left[r(\sin\theta + \frac{\sin 2\theta}{2n}) \right]\omega$$

$$= r\omega\left[\cos\theta + \frac{2\cos 2\theta}{2n} \right]$$

$$= r\omega\left[\cos\theta + \frac{\cos 2\theta}{n} \right]$$

If n is very large;

$$\boxed{a = r\omega^2 \cos\theta}$$ **(as in SHM)**

When θ = 0, at IDC,

$$a = r\omega^2\left(1 + \frac{1}{n} \right)$$

When θ = 180, at 0DC,

$$a = r\omega^2\left(-1 + \frac{1}{n} \right)$$

At θ = 180, when the direction is reversed,

$$a = r\omega^2\left(1 - \frac{1}{n} \right)$$

Angular velocity & angular acceleration of CR (α_c)

$$y = l\sin\phi = r\ \sin\theta$$

$$\sin\ \phi = \frac{\sin\ \theta}{n}$$

Differentiating w.r.t time,

$$\cos\ \phi\ \frac{d\phi}{dt} = \frac{1}{n}\cos\ \theta\ \frac{d\theta}{dt}$$

$$\frac{d\phi}{dt} = \omega_c$$

$$\omega_c = \omega\frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2 - \sin^2\theta}}$$

$$\frac{d\theta}{dt} = \omega$$

$$\cos\phi = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

$$\omega_c = \omega \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$\alpha_c = \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta}\frac{d\theta}{dt}$$

$$= \omega \frac{d}{d\theta}\left[\cos\theta\,(n^2 - \sin^2\theta)^{-\frac{1}{2}}\right]\omega$$

$$= \omega^2\left[\cos\theta \frac{1}{2}(n^2 - \sin^2\theta)^{-\frac{3}{2}}(-2\sin\theta\,\cos\theta) + (n^2 - \sin^2\theta)^{-\frac{1}{2}}(-\sin\theta)\right]$$

$$= \omega^2 \sin^2\theta\left[\frac{\cos^2\theta - (n^2 - \sin^2\theta)}{(n^2 - \sin^2\theta)^{\frac{3}{2}}}\right]$$

$$= -\,\omega^2 \sin\theta\left[\frac{(n^2 - 1)}{(n^2 - \sin^2\theta)^{\frac{3}{2}}}\right]$$

Negative sign indicates that, ϕ reduces (in the case, the angular acceleration of CR is CW)

Engine force Analysis:

Forces acting on the engine are weight of reciprocating masses & CR, gas forces, Friction & inertia forces (due to acceleration & retardation of engine elements)

i) Piston effort (effective driving force)

- Net or effective force applied on the piston.

In reciprocating engine:

The reciprocating parts (masses) accelerate during the first half of the stroke and the inertia forces tend to resist the same. Thus, the net force on the piston is reduced. During the later half of the stroke, the reciprocating masses decelerate and the inertia forces oppose this deceleration or acts in the direction of applied gas pressure and thus effective force on piston is increased.

In vertical engine, the weights of the reciprocating masses assist the piston during out stroke (down) this in creasing the piston effort by an amount equal to the weight of the piston. During the in stroke (up) piston effect is decreased by the same amount.

Force on the piston due to gas pressure; $F_P = P_1\,A_1 - P_2$

P_1 = Pressure on the cover end, P_2 = Pressure on the rod
A_1 = area of cover end, A_2 = area of rod end, m = mass of the reciprocating parts.

Inertia force (F_i) = m a

$$= m.r\omega^2\left(Cos\theta + \frac{Cos2\theta}{n} \right) \qquad \text{(Opposite to acceleration of piston)}$$

Force on the piston F = F_P - F_i

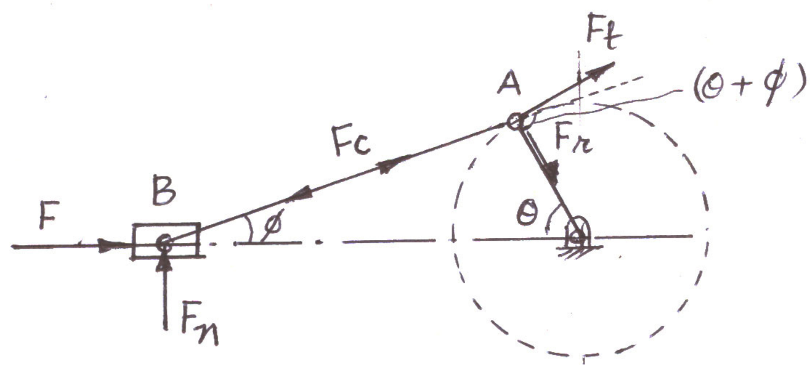
(if F_f frictional resistance is also considered)

$$F = F_P - F_i - F_i$$

In case of vertical engine, weight of the piston or reciprocating parts also acts as force.

$$\therefore\ F = F_P + mg - F_i - F_i$$

ii) Force (Thrust on the CR)



F_c = force on the CR

Equating the horizontal components;

$$F_c\ Cos\phi = F \ \ or \ \ F_c\ \frac{F}{Cos^2\phi}$$

iii) Thrust on the sides of the cylinder

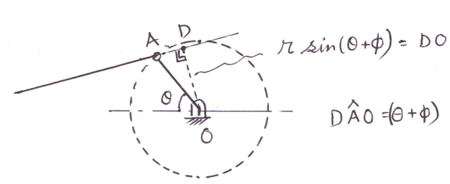
It is the normal reaction on the cylinder walls

$$F_n = F_c \sin\phi = F\tan\phi$$

iv) Crank effort (T)

It is the net force applied at the crank pin perpendicular to the crank which gives the required TM on the crank shaft.

$$F_t \times r = F_c\ r\sin(\theta+\phi)$$



$$F_t = F_c\ \sin(\theta+\phi)$$

$$= \frac{F}{\cos\phi}\sin(\theta+\phi)$$

v) Thrust on bearings (F_r)

The component of F_C along the crank (radial) produces thrust on bearings

$$F_r = F_c\, Cos(\theta+\phi) = \frac{F}{Cos\,\phi}\; Cos(\theta+\phi)$$

vi) Turning moment of Crank shaft

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi}\sin(\theta+\phi)\times r \;=\; \frac{F_r}{\cos\phi}\;(\sin\theta+\cos\phi+\;\cos\theta\sin\phi)$$

$$= F\times r\left(\sin\theta+\cos\theta\,\frac{\sin\phi}{\cos\phi}\right)$$

Proved earlier

$$= F\times r\left(\sin\theta+\cos\theta\,\frac{\sin\theta}{n}\;\frac{1}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}\right) \qquad \cos\phi=\frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$\sin\phi=\frac{\sin\theta}{n}$$

$$= F\times r\left(\sin\theta+\;\frac{\sin 2\,\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

Also,

$$r\sin(\theta+\phi)= OD\;\cos\phi$$

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi}.\; r\;\sin(\theta+\phi)$$

$$= \frac{F}{\cos\phi}.\; OD\;\;\cos\phi$$

$$T = F\times OD.$$

TURNING MOMENT DIAGRAMS AND FLYWHEEL

Introduction:

A flywheel is nothing but a rotating mass which is used as an energy reservoir in a machine which absorbs the energy when the speed in more and releases the energy when the speed is less, thus maintaining the fluctuation of speed within prescribed limits. The kinetic energy of a rotating body is given as ½ $I_0\,\omega^2$, where I_0 is the mass moment of inertia of the body about the axis of rotation and ω is the angular speed of rotation. If the speed should decrease; energy will be given up by the flywheel, and, conversely, if the speed should increase energy will be stored up in the flywheel.

There are two types of machines which benefit from the action of a flywheel. The first type is a punch press, where the punching operation is intermittent Energy is required in spurts and then only during the actual punching operation. This energy can be provided in the following two ways: (i) with a large motor which is capable of providing the energy when required; or (ii) with a small motor and a flywheel, where the small motor may provide the energy to a flywheel gradually during the time when the punching operation is not being carried out. The latter method would definitely be the cheaper and would provide for less sudden drain of power from the power lines to the motor, which is very desirable.

The second type is a steam engine or an internal combustion engine, where energy is supplied to the machine at a non-uniform rate and withdrawn from the engine at nearly a constant rate. Under such a condition, the output shaft varies in speed. The speed increases where there is an excess of supplied energy; and the speed decreases where there is a deficiency of energy. The use of a flywheel would allow the engine to operate with a minimum speed variation because it would act as a reservoir for absorbing the excess energy; during the period when an excess of energy was being supplied, to be redistributed when the energy supplied was not sufficient for the load on the engine. It is evident that, it is not possible to obtain an absolutely uniform speed of rotation of the output shaft if the power is supplied at a variable rate even with a flywheel because a change of speed of the flywheel is necessary to permit redistribution of the energy. However, for a given change of energy in the flywheel, the speed variation may be made very small by using a large mass. Practically, there is no need of using masses any larger than necessary for the proper operation of a given machine. The analysis is aimed to determine the size of flywheel necessary.

Difference between Governor and Flywheel:

A governor controls the speed of the output shaft within close limits, but its action depends upon controlling the amount of working fluid to the engine as required by the load on the engine. The flywheel, on the other hand, serves only to smooth out the energy transfer in each energy cycle. For example, if an engine is operating at quarter load, with the governor in a particular position controlling the amount of working media to the engine; the flywheel would take care of redistributing the energy throughout a cycle. If the load was increased to full load the governor action would permit more working fluid to the engine maintaining the speed of the engine, but when balance of working fluid to the engine and load on the engine was reached, the flywheel would continue its action of redistributing the energy throughout a cycle. Changes of seed in an engine will cause the governor to respond and attempt to do the flywheels job. Usually, the effect of the governor is disregarded in the design of the flywheel. The flywheel analysis is limited to engines receiving power at a variable rate and delivering it to a shaft at an approximately constant rate.

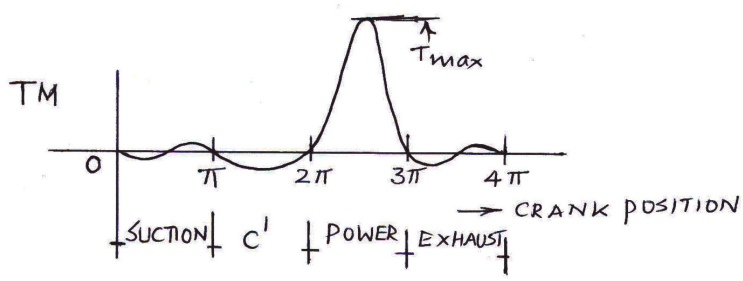
Crank effort diagrams or Turing moment diagrams:

It is the graphical representation of turning moment or crank effort for the various positions of the crank. The TM is plotted on 'y' axis and crank angle on 'x' axis.

Uses of turning moment Diagram

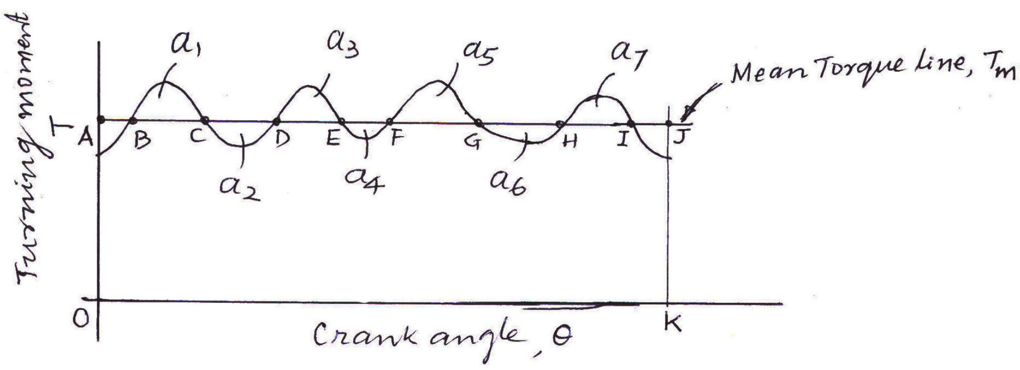
1) The area under the turning moment diagram represents work done per cycle. The area multiplied by number of cycles per second gives the power developed by the engine.
2) By dividing the area of the turning moment diagram with the length of the base we get the mean turning moment. This enables us the find the fluctuation of energy.
3) The max. ordinate of the turning moment diagram gives the maximum torque to which the crank shaft is subjected. This enables us the find diameter of the crank shaft.

TMD for a four stroke I.C. Engine



We know that four stroke cycle internal combustion engine there is one working stroke after the crank has turned through two revolutions (4π or 720°). Since the pressure inside the engine cylinder is less than the atmospheric pressure during suction stroke therefore a negative loop is formed as shown in figure. During compression stroke the work done on engine the gases therefore a higher negative loop is obtained. During expansion or working stroke the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases; therefore a negative loop is firmed.

Fluctuation of energy



The fluctuation of the energy is the excess energy developed by the engine between two crank position or difference between maximum and minimum energies is known as fluctuation of energy. TMD for a multi cylinder engine is as shown in figure. The horizontal line AG represents mean torque line. Let a_1, a_3, a_5 be the areas above the mean torque line a_2, a_4 & a_6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving part of the engine.

Let the energy in the fly wheel at $A = E$
Energy at $B = E + a_1$
Energy at $C = E + a_1 - a_2$
Energy at $D = E + a_1 - a_2 + a_3$
Energy at $E = E + a_1 - a_2 + a_3 - a_4$
Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$
Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

Suppose greatest of these energies is at B and least at E,

Maximum energy in the fly wheel $= E + a_1$
Minimum energy in the fly wheel $= E + a_1 - a_2 + a_3 - a_4$
$\therefore$ Maximum fluctuation of energy (ΔE) = max. energy – min. energy

$$\Delta E = (E + a_1) - (E + a_1 - a_2 + a_3 - a_4)$$
$$\Delta E = a_2 - a_3 + a_4$$

Co-efficient of fluctuation of energy

It may be defined as the ratio of maximum fluctuation of energy to the work done per cycle:

Co-efficient of fluctuation of energy $= \dfrac{\Delta E}{W.D / Cycle}$

$$W.D / Cycle = \frac{P \times 60}{n}$$

Where P= power transmitted
n = number of working strokes/ minute

Fluctuation of energy and speed in Terms of Torques:

The driving torque T produced by an engine (crank effort) fluctuates during any one cycle, the manner in which it varies depending on the type of engine, number of cylinders, etc. It can usually be assumed that, the resisting torque due to the load T_m is constant, and when $T > T_m$ the engine will be accelerating, and vice versa. If there are N complete cycles per minute and n rpm, then the engine power is given by

$$\text{Power} = N \int T \, d\theta = 2\pi \, n \, T_m$$

T_m = mean height of turning moment diagram. For any period during which $T > T_m$, the area cut off on the turning moment diagram represents “excess energy”, which goes to increase the speed of the rotating parts, i.e., excess energy,

$$\Delta E = \int (T - T_m)\, d\theta = \frac{1}{2} I_0 (\omega_{\max}^2 - \omega_{\min}^2) \text{ same as before.}$$

In simple cases, ΔE is given by the area of one “loop” intercepted between T and T_m but for a multi cylinder engine a further analysis is necessary.

Coefficient of Fluctuation of speed:

The coefficient of fluctuation of speed is defined as

$$\delta = \frac{\omega_{\max} - \omega_{\min}}{\omega_{mean}}$$

Where $\omega_{\max}$ = max. angular speed of the flywheel

$\omega_{\min}$ = min. angular speed of the flywheel

ω_{mean} = average angular speed of the flywheel

or

$$\delta = \frac{V_{\max} - V_{\min}}{V_{mean}}$$

The maximum permissible coefficients for different applications are as follows:

$\delta = 0.2$ for pumps, crushing machines

= 0.003 for alternating current generators.

In general, δ varies between the above values for all machines.

Weight of a flywheel for given value of δ :

The kinetic energy (K.E.) of a body rotating about a fixed centre is,

$$\text{K.E} = \frac{1}{2} I_0\ \omega_{mean}^2$$

The maximum fluctuation of K.E ΔE is given by

$$\Delta E = \frac{1}{2} I_0\ (\omega_{\max}^2 - \omega_{\min}^2)$$

Multiply and divide by r^2 on the right hand side, we have

$$\Delta E = \frac{I_0}{2r^2}\,(r\omega_{\max})^2 - (r\omega_{\min})^2$$

Where r is the mean radius of the flywheel rim.

$$\therefore \Delta E = \frac{I_0}{2r^2}\;(V_{\max}^2 - V_{\min}^2) = \frac{I_0}{2r^2}\;(V_{\max} + V_{\min})\;(V_{\max} - V_{\min})$$

but, $V_{mean} = \dfrac{V_{\max} + V_{\min}}{2}$ *and* $\delta = \dfrac{V_{\max} - V_{\min}}{V_{mean}}$

we have, $\Delta E = \dfrac{I_0}{2r^2}\;(2V_{mean})(V_{mean}\delta) = \dfrac{I_0 V_{mean}^2 \delta}{r^2} = \dfrac{(mk^2)V_{mean}^2\delta}{r^2} = mk^2\,\omega^2\,\delta$

It is usual practice, in flywheel analysis, to consider the mass of the flywheel concentrated at the mean radius of the rim, and to make corrections later for the fact that the arms and hubs contributed to the flywheel effect. That is, k is assumed to be equal to *r*, the mean radius of the rum.

$$\therefore \Delta E = (m\,V_{mean}^2\delta) = mr^2\,\omega^2\,\delta$$

or the mass of the flywheel, $m = \dfrac{(\Delta E)}{V_{mean}^2\delta}$

$$m = \frac{2(\Delta E)}{V_{mean}^2\delta} \quad \text{for flat circular plate.}$$

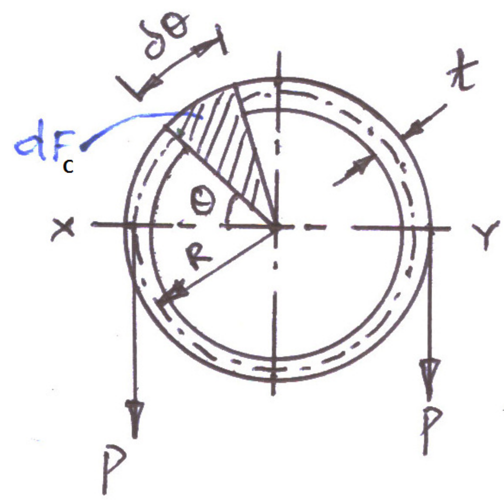
For a solid disc of diameter D, $k^2 = \dfrac{D^2}{8}$ and for ring or rim of diameters D and d, $k^2 = \left(\dfrac{D^2 + d^2}{8}\right)$

Notice that, as a result of the above assumption, the actual mass of the rim of the flywheel may be taken as approximately 10% less than that calculated by the above formula to allow for the effect of the arms and hub of the flywheel and other rotating parts, which is sufficient for the usual designs encountered.

For a given engine with a flywheel of a given material, the safe allowable mean rim velocity V_{mean} is determined by the material and the centrifugal stresses set in the rim. Consequently, with a velocity established for a given type of flywheel, the δ set by the type of application. The problem now is to find the maximum excess or deficiency of energy (ΔE), during an energy cycle which causes the speed of the flywheel to change from $V_{\max}$ to $V_{\min}$ or vice versa.

Size of fly wheel and hoop stress developed in a fly wheel.

Consider a rim of the fly wheel as shown in figure. Let D = mean diameter of rim, R = mean radius of rim, *t* = thickness of the fly wheel, A = cross sectional as area of rim in m^2 and ρ be the density of the rim material in Kg/m^3, N be the speed of the fly wheel in rpm, ω= angular velocity in rad/sec, V = linear velocity in m/σ, hoop stress in N/m^2 due to centrifugal force.



Consider small element of the rim. Let it subtend an angle $\delta\theta$ at the centre of flywheel.
Volume of the small element $= R\delta\theta.A$.
Mass of the small element $= dm = R\delta\theta.A\ \rho$

The centrifugal force on the small element

$$dF_C = dm\omega^2 R$$
$$= R\delta\theta.A\omega^2 R\ \rho$$
$$= R^2 A.\omega^2 \delta\theta\ \rho$$

Resolving the centrifugal force vertically

$$dF_C = dF_C Sin\theta$$

$$=\rho\ R^2 A\omega^2 Sin\theta.\ \delta\theta \quad ---(1)$$

Total Vertical upward force across diameter X & Y

$$= \int_0^{\pi} \rho R^2 A\omega^2 Sin\theta.\ \delta\theta$$
$$= \rho R^2 A\omega^2 \int_0^{\pi} Sin\theta.\ \delta\theta$$
$$2\rho\ = 2\rho A R^2 \omega^2$$

This vertical upward force will produce tensile stress on loop stress developed & it is resisted by 2P.

We know that, $\sigma = P / A$

$$P = \sigma A$$

$$\therefore\ 2P = 2\sigma A$$

$$PAR^2\omega^2 = 2\sigma A$$

$$\boxed{\sigma = \rho R^2 \omega^2}$$ % up to this deviation

Also,

Linear velocity V=Rx ω

$$\sigma = \delta V^2$$

$$\boxed{V = \sqrt{\sigma / \delta}}$$

Mass of the rim = volume x density

$$\boxed{m = \pi\, dA \times \rho}$$

Problem 1:

A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of the machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during ½ revolution and remains constant fore the following revolution. It then falls uniformly to 750 N-m during the next ½ revolution and remains constant for one revolution, the cycle being repeated thereafter.

Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 mm.

Solution.
Given: $N = 250\ r.p.m\ \ or\ \omega = 2\pi \times 250/60\ = 26.2\,rad/s;\ m = 500kg;\ k = 600\ \ m = 0.6$

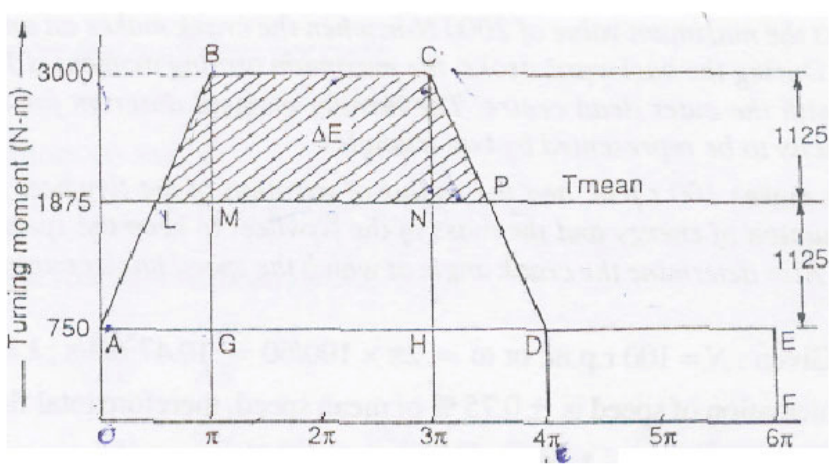
The turning moment diagram for the complete cycle is drawn.

The torque required for one complete cycle

$$= Area\ of\ \ figure\ OABCDEF$$
$$= Area\ \ OAEF + Area\ ABG + Area BCHG + Area\ CDH$$
$$= OF \times OA + \frac{1}{2} \times AG \times\ BG\ +\ GH\ \times\ CH + \ \frac{1}{2} \times HD \times CH$$
$$= 6\pi \times 750 + \frac{1}{2} \times \pi(3000 - 750) + 2\pi(3000 - 750) + \ \frac{1}{2} \times \pi(3000 - 750)$$
$$= 11250\ \pi\, N - m$$

Torque required for one complete cycle $= T_{mean} \times \pi\, N - m$

$$\therefore T_{mean} = 11250\pi / 6\pi = 1875\ N - m$$



Power required to drive the machine, $P = T_{mean} \times \omega = 11875\ \times 26.2 = 49125W = 49.125kW$.

To find Coefficient of fluctuation of speed, δ.

Find the values of *LM* and *NP*.

From similar triangles *ABG* and *BLM*,

$$\frac{LM}{AG} = \frac{BM}{BG} \quad or \quad \frac{LM}{\pi} = \frac{3000-1875}{3000-750} = 0.5 \quad or \quad LM = 0.5\pi$$

From similar triangles *CHD* and *CNP*,

$$\frac{NP}{HD} = \frac{CN}{CH} \quad or \quad \frac{NP}{\pi} = \frac{3000-1875}{3000-750} = 0.5 \quad or \quad NP = 0.5\pi$$

From the figure, we find that,

$BM = CN = 3000 - 1875 = 1125$ N-m

The area above the mean torque line represents the maximum fluctuation of energy. Therefore the maximum fluctuation of energy, ΔE

= Area *LBCP* = Area *LBM* + Area *MBCN* + Area *PNC*

$$= \frac{1}{2} \times LM \times BM + MN \times BM + \frac{1}{2} \times NP \times CN$$

$$= \frac{1}{2} \times 0.5\,\pi \times 1125 + 2\pi \times 1125 + \frac{1}{2} \times 0.5\,\pi \times 1125 = 8837\ N-m$$

We know that maximum fluctuation of energy (ΔE),

$$8837 = m.k^2.\omega^2.\delta = 500\,(0.6)^2\,(26.2)^2\,\delta = 123\,559\,\delta$$

$$\boxed{\delta = 0.071}$$

Problem 2

The torque delivered by two stroke engine is represented by T = 1000+300 sin 2θ-500 cos 2θ where θ is angle turned by the crack from inner dead under the engine speed. Determine work done per cycle and the power developed.

Solution

θ, deg.	$T, N-m$
0	500
90	1500
180	500
270	1500
360	500

Work done / cycle = Area under the turning moment diagram.

$$= \int_0^{2\pi} T\, d\theta$$

$$= \int_0^{2\pi} (1000 + 300 \sin 2\theta - 500 \cos 2\theta)\, d\theta$$

$$= 2000\pi\ N-m$$

$$T_{mean} = \frac{W.D\ / cycle}{2\pi}$$

$$= \frac{2000\pi}{2\pi} = 1000\ N-m$$

Power developed $= T_{mean} \times \omega_{mean}$

$$= 1000 \times \frac{2\pi\ N}{60}$$

$$= 1000 \times \frac{2\pi \times 200}{60}$$

$$= 26179 W$$

Problem: 3

The turning moment curve for an engine is represented by the equation,

$T = (20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta)$ N-m, where θ is the angle moved by the crank from inner dead centre. If the resisting torque is constant, find:

1. Power developed by the engine;
2. Moment of inertia of flywheel in kg-m^2, if the total fluctuation of speed is not the exceed 1% of mean speed which is 180 r.p.m. and
3. Angular acceleration of the flywheel when the crank has turned through 45° from inner dead centre.

Solution:

Given, T = (20 000 + 9500 sin 2θ - 5700 cos 2θ) N-m ;

N = 180 r.p.m. or ω = 2π x 180/60 = 18.85 rad/s

Since the total fluctuation of speed (ω_1-ω_2) is 1% of mean speed (ω), coefficient of fluctuation of speed,

$$\delta = \frac{\omega_1 - \omega_2}{\omega} = 1\% = 0.01$$

1. Power developed by the engine.

Work done per revolution

$$= \int_0^{2\pi} T d\theta = \int_0^{2\pi} (20000 + 9500 \sin 2\theta - 5700 \cos 2\theta)\, d\theta$$

$$= \left[20000\, \theta - \frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2} \right]_0^{2\pi}$$

$$= 20000 \times 2\pi = 40\,000\, \pi \;\; N-m$$

Mean resisting torque of the engine,

$$T_{mean} = \frac{Work\ done\ per\ revolution}{2\pi} = \frac{40\,000\, \pi}{2\pi} = 20000 \;\; N-m$$

Power developed by the engine

$$= T_{mean} . \omega = 20\,000 \times 18.85 = 377\,000 W = 377\, kW.$$

2. Moment of inertia of the flywheel

The turning moment diagram for one stroke (i. e. half revolution of the crankshaft) is shown in the Fig. Since at points B and D, the torque exerted on the crankshaft is equal to the mean resisting torque on the flywheel, therefore,

$$T = T_{mean}$$

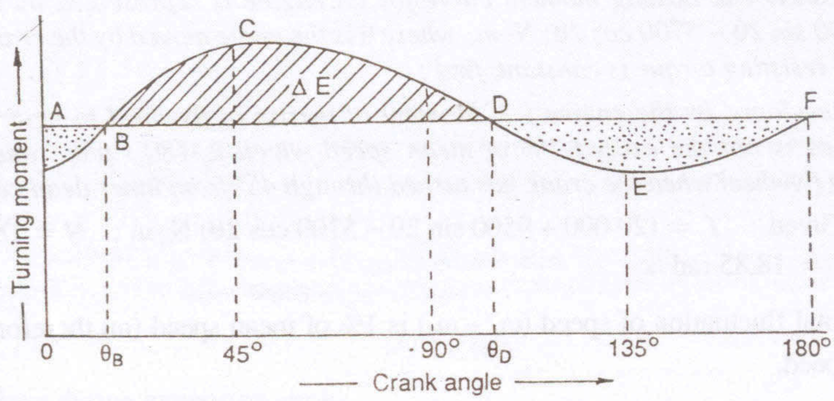
$$20\,000 + 9500 \sin\, 2\theta - 5700\;\; \cos\, 2\theta - 20\,000$$

or $$9500 \sin\, 2\theta = 5700\;\; \cos\, 2\theta$$

$$\tan\;\, 2\theta = \sin\, 2\theta \,/ \cos\, 2\theta = \;5700/9500 = 0.6$$

$\therefore$ $$2\theta = 31° \;\; or \theta = 15.5°$$

$\therefore$ $$\text{i.e., } \theta_B \; = 15.5° \;\; and \;\; \theta_D \; = 90° + 15.5° = 105.5°$$



Maximum fluctuation of energy,

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean}) d\theta$$

$$= \int_{15.5°}^{105.5°} (20000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\ 000) d\theta$$

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean}) d\theta = \left[-\frac{9500 \sin 2\theta}{2} - \frac{5700 \cos 2\theta}{2} \right]_{15.5°}^{105.5°} = 11078\ N - m$$

Maximum fluctuation of energy (ΔE),

$11\ 078 = I.\omega.\ \delta = I(18.85)^2\ 0.01 = 3.55\ I$

$I = 11078/3.55 = 3121$ kg-m^2.

3. Angular acceleration of the flywheel

Let α = Angular acceleration of the flywheel, and

θ = Angle turned by the crank from inner dead centre = 45°... (Given)

The angular acceleration in the flywheel is produced by the excess torque over the mean torque. Excess torque at any instant,

$$T_{excess} = T - T_{mean}$$

$$20\ 000 + 9500 \sin\ 2\theta - 5700\ \cos\ 2\theta = 20\ 000$$

$$9500 \sin\ 2\theta - 5700\ \cos\ 2\theta$$

$\therefore$ Excess torque at 45°= 9500 sin 90° - 5700 cos 90° =9500Nm

We also know that excess torque= $I.\alpha$ = 3121 xα

From equations (i) and (ii),

α = 9500 / 3121 = 3.044 rad/s^2.

Problem 4

The torque exerted on the crankshaft is given by the equation

$$T_m = 1500 + 240 \sin 2\theta - 200 \cos 2\theta\ Nm.$$

Where θ is the crank angle displacement from the inner dead centre. Assuming the resisting torque to be constant, determine (a) the power of the engine when the speed is 150 rpm (b) the moment of inertia of the flywheel if the speed variation is not to exceed ±0.5% of the mean speed and (c) the angular acceleration of the flywheel when the crank has turned through 30° from the inner dead center.

SOLUTION: (a) Since the fluctuating terms sin2θ and cos2θ have zero mean, we have

$T_m = 1500\ Nm$

$\therefore$ Power of the engine $= \frac{2\pi}{60} n\, T_m$

$= \frac{2\pi \times 150 \times 1500}{60}$

= 23.5 kW

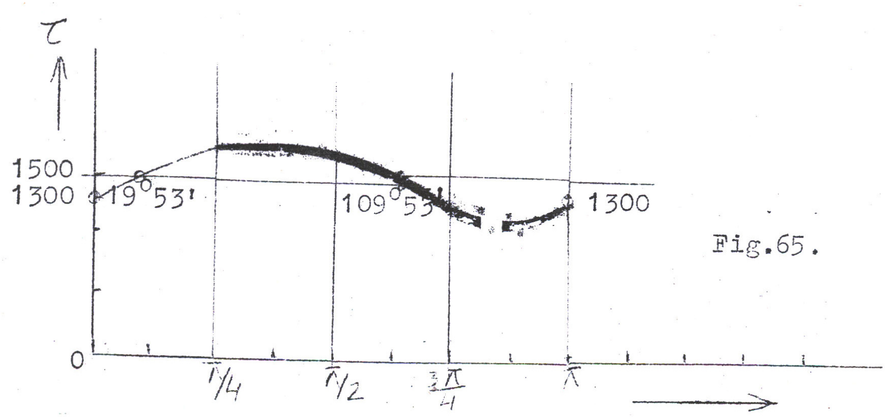


Fig.65.

(b) $T = T_m$ when tan 2θ $= \frac{200}{240} = 0.8350$

$$\therefore\ 2\theta = 39°46' \ \ or\ \ 180 + 39°46'$$

$$\theta = 19°53' \ \ and\ \ 109°53'$$

and, excess energy, $$\Delta E = \int_{109°53'}^{109°53'} (240 \sin 2\theta - 200 \cos 2\theta)\, d\theta$$

= 314Nm.

Speed variation $\pm 0.5\% = \pm \frac{\Delta E}{2I\omega_{mean}} \times 100$

$$\therefore\ I = \frac{314 \times 100}{\left(\frac{150\pi}{30}\right)} = 126.5 Nm^2$$

(c) At θ = 30°, accelerating torque

$T - T_m = 240 \times 0.866 - 20 \times 0.5$

= 108 Nm.

$\therefore$ Angular acceleration, $\alpha = \frac{108}{I} = 0.855\ rad/\sec^2$

Problem 5: The equation of the turning moment diagram of a three crank engine is 21000+7000 sin3θ Nm. Where θ in radians is the crank angle. The moment of inertia of the flywheel is $4.5 \times 10^3\ Nm^2$ and the mean engine speed is 300 rpm. Calculate the power of the engine and the total percentage fluctuation of speed of the flywheel (i) if the resisting torque is constant (ii) if the resisting torque is 21000 + 3000 sinθ Nm.

a) $T_m = 21000\,Nm.$

$$\text{Power} = \frac{2\pi \times 21000 \times 300}{60} = 660\,kW.$$

b) (i) $\Delta E = \int_0^{\frac{\pi}{3}} 7000 \sin 3\theta d\theta = 4666.7\ Nm.$

$$\therefore \text{Total percent fluctuation of speed} = \frac{100\ \Delta E}{I\omega^2_{mean}}$$

$$= \frac{100 \times 4666.7 \times 9.8}{45 \times 10^3 \times \left(\frac{300\pi}{30}\right)^2}$$

$$= 1.04\%$$

(ii) Engine torque = load torque, at crack angles given by

7000 sin3θ = 3000 sinθ

i.e., 2.33 (3sinθ - 4 $\sin^3$θ) = sinθ

One solution is sinθ =0, i.e., θ = 0 and 180º, and the other is sinθ = ±0.803, i.e., θ = 53°24'or 126°36' between 0° and 180º. The intersections are shown in figure and the areas between the curves represent increase or decrease of total energy. The numerically longest is between θ = 53° 24' and 126°36'.

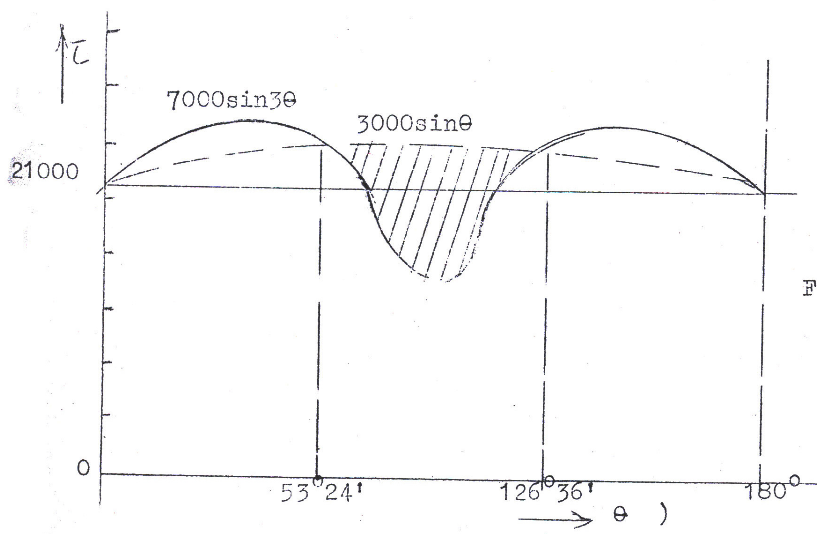


Fig.66.

$$i.e.,\ \Delta E = \int_{53°24'}^{126°36'} (7000 \sin 3\theta - 3000 \sin \theta)\, d\theta$$

=7960 Nm.

Therefore, the total (percentage) fluctuation of speed $\frac{100\ \Delta E}{I\omega^2_{mean}}$

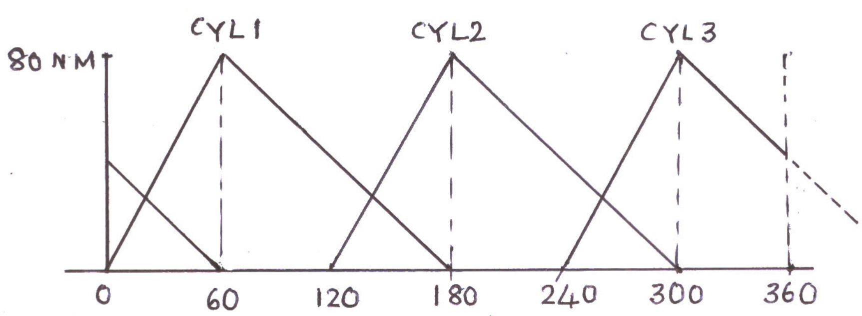
$$= \frac{100 \times 7960 \times 9.8}{4.5 \times 10^3 \times \left(\frac{300\pi}{30}\right)^2}$$

= 1.65%

Problem 6:

A 3 cylinder single acting engine has its cranks set equally at 120° and it runs at 600 rpm. The Torque crank angle diagram for each cylinder is a triangle for the power with maximum torque 80 N-m at 60° after dead centre of the corresponding crank. The torque on the return stroke is sensibly zero.

Determine the (a) Power developed
(b) K if the flywheel used has a mass of 10 Kg. and radius of gyration is 8 cm
(c) Coefficient of fluctuation of energy
(d) Maximum angle of the flywheel



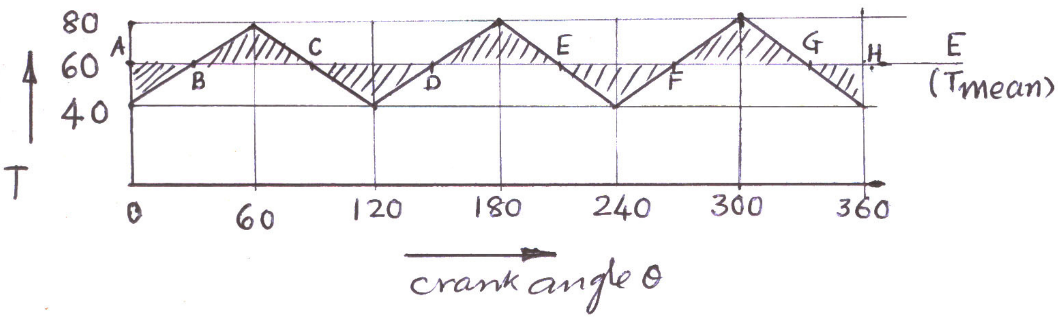
Work done / cycle = Area of 3 triangles

$$3 \times \frac{1}{2}\pi \times 80 = 120\pi\ N-m$$

(a) Power developed $= \frac{work\ done\ /cycle \times cycle\ /\min}{60 \times 1000} kW$

$$= \frac{120 \times \pi \times 600}{60 \times 1000} = 3.75\ kW$$

(b) $T_{mean} = \frac{work\ done\ /cycle}{crank\ angle\ /\ cycle} = \frac{120\ \pi}{2\ \pi} = 60\ N-m$



Energy at $A = E$

Energy at $B = (E - \frac{1}{2}.\frac{\pi}{6}.20) = \frac{10\pi}{6}$

Energy at $C = (E - \frac{10\pi}{6} + \frac{1}{2}.\frac{\pi}{3}.20) = E + \frac{10\pi}{6}$

Energies at D, E, F, G & H will be,

$$E - \frac{10\pi}{6}, E + \frac{10\pi}{6}, E - \frac{10\pi}{6}, E + \frac{10\pi}{6} \; E \; respectively$$

$$\Delta E = \left(E + \frac{10\pi}{6}\right) - \left(E - \frac{10\pi}{6}\right) = \frac{10\pi}{3} \; N-m$$

Maximum Fluctuation of energy	Maximum energy	Minimum energy

$$\omega = \frac{2\pi \; 600}{60} = 20\,\pi \; rad/s$$

$$\Delta E = I\,\omega^2\,\delta = I\,\omega\,(\omega_1 - \omega_2)$$

$$= mk^2\,\omega\,(\omega_1 - \omega_2)$$

$$(\omega_1 - \omega_2) = \frac{\Delta E}{mk^2\omega} = \frac{10\pi}{3} \times \frac{1}{10 \times (0.08)^2\,20\pi} = 2.604 \; rad/s$$

$$\delta = \frac{\omega_1 - \omega_2}{\omega} = \frac{2.604}{20\,\pi} \times 100 = 4.44\%$$

(c) Coefficient of fluctuation of energy $= \frac{Maximum\;fluctuation\;of\;energy}{work\;done/cycle}$

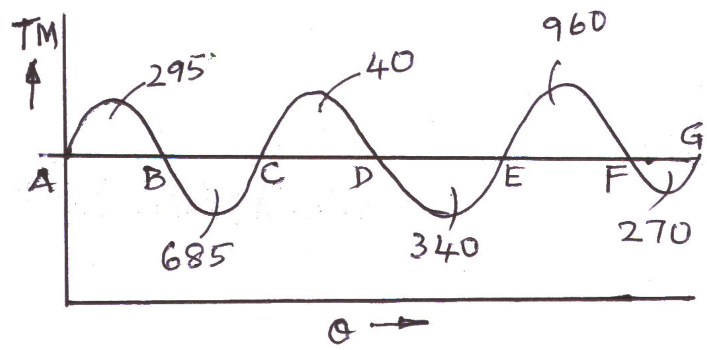
$$= \frac{10\,\pi}{3}.\frac{1}{120\,\pi} = 0.0278 \; or \; 2.78\%$$

(d) $T_{max} - T_m = I\alpha$

$$\therefore \alpha = \frac{T_{max} - T_m}{I} = \frac{80-60}{10\times(0.08)^2} = 312.5\ rad/s^2$$

Problem 7:

The TMD for a petrol engine is drawn to the following scale, turning moment, 1mm = 5Nm, crank 1mm = 1°. The TMD repeats itself at every half revolution of the engine & areas above & below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm^2. The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150mm. Calculate the maximum fluctuation of energy & co-efficient of fluctuation of speed when engine runs at 1800rpm



Energy at $A = E$
Energy at $B = E + a_1$
$= E + 295$
Energy at $C = E + 295 - 685 = E - 390$
Energy at $D = E + 295 - 685 + 40 = E - 350$
Energy at $E = E - 350 - 340 = E - 690$
Energy at $F = E - 690 + 960 = E + 270$
Energy at $G = E + 270 - 270 = E$
$\therefore A = G$

Max Energy $= E + 295$
Min Energy $= E - 690$

Maximum Fluctuation of Energy $\Delta E = E + 295 - (E - 690)$

$= 985 mm^2$

Scale: $1mm = 5Nm$ & $1mm = 1°$

$$Torque \times \theta = \frac{5}{180}\pi \times 1 = \frac{\pi}{36} Nm$$

$$\Delta E = 985 \times \frac{\pi}{36} = 85.95 Nm$$

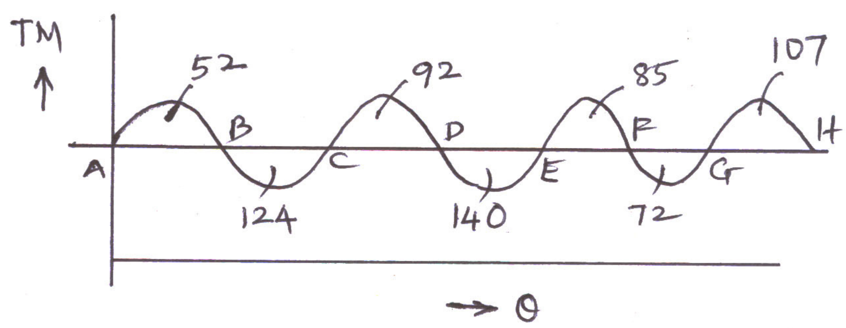
$m = 36kg, k = 150mm, \quad N = 1800rpm$

$$\Delta E = mk^2\omega^2\delta$$
$$86 = 36\times 0.15^2 \times \left(\frac{2\Pi(1800)}{60}\right)^2 \delta$$
$$\delta = 0.003 \quad or \quad 0.3\%$$

Problem 8:

The turning moment diagram for a multi cylinder engine has been drawn to a scale 1mm = 600 Nm vertically and 1mm = 3° horizontally. The intercepted areas between the output torque curve and mean resistance line taken in order from one end are as follows + 52, -124, +92, -140; 85, -72 and 107 mm^2 when the engine is running at a speed of 600 rpm. If the total fluctuation of speed is not exceed 1.5% of the mean, find the necessary mass of the fly wheel of radius 0.5 m.

Solution:



$N = 600 \quad rpm$

Co-efficient of fluctuation of speed, $\delta = \dfrac{\omega_1 - \omega_2}{\omega} = 1.5 + 1.5 = 3\%$

$\Delta E = mR^2\omega^2 \delta$

Energy at $A = E$

Energy at $B = E + 52$

Energy at $C = E + 52 - 124 = E - 702$

Energy at $D = E - 72 + 92 = E + 20$

Energy at $E = E + 20 - 140 = E - 120$

Energy at $F = E - 120 + 85 = E - 35$

Energy at $G = E - 35 - 72 = E - 107$

Energy at $H = E - 107 + 107 = E$

$\Delta E = E + 52 - (E - 120) = 172 \; mm^2$

Scale: $T\theta = 1 \; mm^2 = 600 \times 3 \times \dfrac{\pi}{180} = 31.41 \; Nm$

$$\Delta E = 172 \times 31.41 = 5402.52 \, Nm$$

$$\Delta E = mR^2\,\omega^2\,\delta$$

$$5402.5^2 = m(0.5)^2\left(\frac{2\pi\times 600}{60}\right)^2\times\frac{3}{100}$$

$$m = 182.47\ kg$$

Problem 9:

The TMD for a multi cylinder engine has been drawn to a scale 1mm to 500Nm torque & 1mm to 6° of crank displacement. The intercepted area in order from one end is mm^2 are -30, 410, -280, 320, -330, 250, -360, +280, -260 mm^2 when engine is running at 800rpm. The engine has a stroke of 300mm & fluctuation of speed is not to exceed ±2% of the mean speed, determine

1. a suitable diameter & cross section of the fly wheel rim for a limiting value of the safe centrifugal stress of 7MPa. The material density may be assumed as 7200 kg/m^3. The width of the rim is to be 5times the thickness.

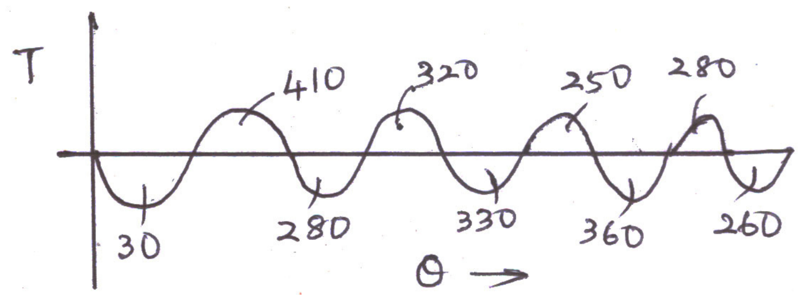
   Solution:

$N = 800\ \ rpm$

$\pm\ 2\ \%means,\ \delta = 4\% = 0.04$

$\sigma = 7\,Mpa = 7\ \ N/m2$

$\rho = 7\,200\ kg/m^3$



Energy at $A = E$
Energy at $B = E - 30$
Energy at $C = E - 30 + 410 = E + 380$
Energy at $D = E + 380 - 280 = E + 100$
Energy at $E = E + 100 + 320 = E + 420$
Energy at $F = E + 420 - 330 = E + 90$
Energy at $G = E + 90 + 250 = E + 340$
Energy at $H = E + 340 - 360 = E - 20$
Energy at $I = E - 20 + 280 = E + 260$
Energy at $J = E + 260 - 260 = E$

$$\Delta E = E + 420 - (E - 30)$$
$$= 450mm^2$$

$1mm = 500Nm$, $1mm = 6°\ (0.1047\ radians)$, $1mm^2 = 52.35Nm$

$\Delta E = 450\times 52.35 = 23557.5$ Nm

$\sigma = \rho V^2$ $\qquad\qquad \Delta E = mr^2\,\omega^2\delta$

$7\times 10^6 = 7200\ V^2$ $\qquad = mV^2\delta$

$V = r\omega$

$V = 31.18\ m/s$

$$V = \frac{\pi DN}{60}, \; D = 0.745\,m$$

Cross sectional area $A = bt$

$A = (5t)t = 5t^2$

Fluctuation of energy $\Delta E = mV^2\delta$

$$23.56 \times 10^3 = m(31.18)^2 \; (0.04)$$

$$m = 605 \; kg$$

$m = Volume \times Density$

$\pi DA \times \rho$

$605 = \pi(0.745)(5t^2)7200$

$t = 0.084\,m$

Area $= 5t^2 = 0.035m^2$

Problem 10:

The T M diagram for a multi cylinder engine has been drawn to a scale of 1 cm = 5000 N-m and 1 cm = 60° respectively. The intercepted areas between output torque curve and mean resistance line is taken in order from one end are – 0.3, +4.1, -2.8, +3.2, -3.3, + 2.5, - 3.6, +2.8, - 2.6 cm^2 , when the engine is running at 800 rpm. The engine has a stroke of 300 mm and fluctuation of speed is not to exceed 2% of the mean speed. Determine a suitable diameter & cross section of the flywheel rim for a limiting value of the shaft centrifugal stress of $280 \times 10^5 \; N/m^2$. The material density can be assumed as 7.2 gm / cm^3. Assume the thickness of the rim to be ¼ of the width.

$$E_{\max} = E + 4.2$$

$$E_{\min} \; E - 0.3$$

$$(\Delta E) = 4.5cm^2 \; or$$

$$= 4.5 \times 5000 \times \frac{\pi}{3} = 23{,}562 \quad N-m$$

$$\Delta E = I\omega^2\delta$$

$$I = \frac{\Delta E}{\omega^2\delta} = \frac{23.562}{\left(\frac{2\pi\,800}{60}\right)^2 0.02} = 168\; kg\,m^2$$

Safe peripheral velocity is given by;

$f = \rho v^2 \ N/m^2$ — f = safe stress N/m^2

or $V = \sqrt{\frac{f}{\rho}} \ m/s$ — V= velocity m/s (peripheral)

ρ = density Kg/m^3

$$= \sqrt{\frac{28 \times 10^5}{7.2 \times \frac{10^6}{1000}}} = 62.36 \ m/s$$

$$\Delta E = I\omega^2 \delta$$

$$\delta = \frac{\Delta E}{I\omega^2} = \frac{\Delta E}{2 \times \frac{1}{2} I\omega^2} = \frac{\Delta E}{2(KE)}$$

$$(KE) = \frac{\Delta E}{2\delta}$$

Also, $V = \frac{\pi DN}{60}$

$$\therefore = \frac{\pi DN}{60} = 62.36 \ ;$$

$$D = 1.4887 \, m$$

Energy of the flywheel $(KE) = \frac{\Delta E}{2\delta} = \frac{23562}{2 \times 0.02} = 589050 \ N - m$

But $KE = \frac{1}{2} mV^2$

$$\therefore \ 589050 \ = \frac{1}{2} m (62.36)^2$$

$$\therefore m = 303 Kg.$$

Also $m = \pi DA\rho$

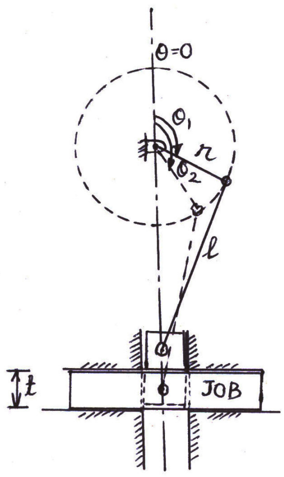
or $A = \frac{m}{\pi D \rho} = \frac{303}{\pi \times 1.4887 \times \frac{7.2 \times 10^6}{1000}} = 89.98 \ cm^2$

Area of cross section $A = t \times b = t \times 4t = 4t^2 = 89.89 \, cm^2$

$$t = \sqrt{\frac{89.98}{4}} = 4.75 \ cm$$

$$b = 4 \times 4.75 = 19 \, cm$$

Flywheel in punching press / Riveting machine



(a) Crank is driven by motor for which supplies a uniform torque.

(b) Load acts from $\theta = \theta_1$ *to* $\theta = \theta_2$ (during Punching). Load is zero for the remaining period.

(c) If flywheel is not there speed increases from $\theta = \theta_2$ *to* $\theta = 2\pi$ $(=0)$ and again from $\theta = 0$ *to* $\theta = \theta_1$

(d) From θ_1 *to* θ_2 big drop in speed.

(e) Use flywheel of suitable I for uniform speed

Let, E = Energy required for one punch

E is determined by Size of the hole, thickness of the blank to be punched and Material property

For stable operation (constant speed), energy supplied to the crank / rev = E (assuming 1 punch / revolution)

Energy supplied to the crank shaft from motor during punching = $E\left[\frac{(\theta_1 - \theta_2)}{(2\pi)}\right]$, if crank rotation is constant (when flywheel is there it is possible)

i.e., $E\left[1 - \frac{(\theta_1 - \theta_2)}{(2\pi)}\right]$ is supplied by flywheel by the decrease in its E_k (Kinetic energy) when the speed falls from ω_{max} *to* ω_{min}

$$\therefore (\Delta E_k)_{max} = E\left[1 - \frac{(\theta_1 - \theta_2)}{(2\pi)}\right] = \frac{1}{2} I(\omega_{max}^2 - \omega_{min}^2) = I\omega^2\delta \text{ (same as before)}$$

θ_1 and θ_2 can be computed only if l, t, r and relative position of job w.r.t. crank shaft are given.

In the absence of data assuming (taking velocity of tool to be constant),

$$\frac{(\theta_2 - \theta_1)}{(2\pi)} \approx \frac{t}{2S} = \frac{t}{4r}$$

S = stroke of the punch $= 2r$

Problem.1

A machine punching 3.8 cm dia hole in a 3.2 cm thick plate does 600 J of work / sq. cm of sheared area. The punch has a stroke of 10.2 cm and punches 6 holed / min. The maximum speed of the flywheel at its radius of gyration is 27.5 m/s. Find the mass of the flywheel so that its speed at the same radius does not fall below 24.5 m/s. Also determine the power of the motor, driving this machine.

d = 3.8cm, t = 3.2 cm, A= 38.2 cm^2

Energy required / punch = 600 x 38.2 = 22.920 J

Assuming, $\frac{(\theta_2 - \theta_1)}{(2\pi)} = \frac{t}{2S} = \frac{3.2}{20.4}$

$$\therefore (\Delta K_E)_{\max} = E\left[1 - \frac{t}{2S}\right] = \frac{1}{2} I\left(\omega_{\max}^2 - \omega_{\min}^2\right)$$

$$= 22.920\left[1 - \frac{3.2}{20.4}\right] = \frac{1}{2} mk^2\left(\omega_{\max}^2 - \omega_{\min}^2\right)$$

$$V_{\max} = k\ \omega_{\max} = 27.5 m/s$$

$$V_{\min} = k\ \omega_{\min} = 24.5 m/s$$

We get,

$$22920\left[1 - \frac{3.2}{20.4}\right] = \frac{1}{2} m\left(27.5^2 - 24.5^2\right) = \frac{1}{2} m\ 158$$

$$\therefore\ m = 244 kg.$$

The energy required / minute is $6 \times 22920 J$

$$\therefore\ Motor\ \ power\ = \frac{6 \times 22920}{1000 \times 60}\ k\omega = 2.292 kW$$

Problem.2

A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs 10 000 N-m of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find the speed immediately after riveting. How many rivets can be closed per minute.

Solution.

Given: $P = 3\ kW$; $m = 150\ kg$; $k = 0.6$m; $N_1 = 300$ r.p.m. or $\omega_1 = 2\pi \times 300/60 = 31.42$rad/s

Speed of the flywheel immediately after riveting

Let ω_2 = Angular speed of the flywheel immediately after riveting.

We know that, energy supplied by the motor,

$$E_2 = 3\ kW = 3000\ W = 3000\ N-m/s \qquad (\because 1W = 1\ N-m/s)$$

But, energy absorbed during one riveting operation which takes 1 second,

$$E_1 = 10\ 000\ N-m$$

$\therefore$ Energy to be supplied by the flywheel for each riveting operation per second or the maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 10\ 000 - 3000 = 7000 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$7000 = \frac{1}{2} \times m.k^2 \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times 150\ (0.6)^2 \left[(31.42)^2 - (\omega_2)^2\right]$$

$$= 27\left[987.2 - (\omega_2)^2\right]$$

$$\therefore \qquad (\omega_2)^2 = 987.2 - 7000/27 = 728 \ or\ \omega_2 = 26.98\ rad/s$$

Corresponding speed in r.p.m.,

$$N_2 = 26.98 \times 60 / 2\pi = 257.6\ r.p.m.$$

Number of rivets that can be closed per minute.

Since, the energy absorbed by each riveting operation which takes 1 second is 10 000 N-m, therefore number of rivets that can be closed per minute,

$$= \frac{E_2}{E_1} \times 60 = \frac{3000}{10\ 000} \times 60 = 18\ rivets$$